CHAPTER 1
Preparation for Calculus

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C H A P T E R 1
Preparation for Calculus

Section 1.1 Graphs and Models

1. $y = -\frac{3}{2}x + 3$

x-intercept: $(2, 0)$

y-intercept: $(0, 3)$

Matches graph (b).

2. $y = \sqrt{9 - x^2}$

x-intercepts: $(-3, 0), (3, 0)$

y-intercept: $(0, 3)$

Matches graph (d).

3. $y = 3 - x^2$

x-intercepts: $(\sqrt{3}, 0), (-\sqrt{3}, 0)$

y-intercept: $(0, 3)$

Matches graph (a).

4. $y = x^3 - x$

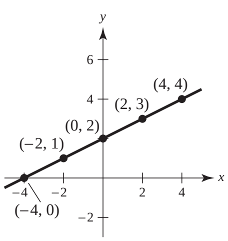
x-intercepts: $(0, 0), (-1, 0), (1, 0)$

y-intercept: $(0, 0)$

Matches graph (c).

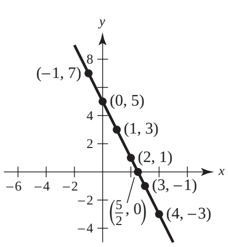
5. $y = \frac{1}{2}x + 2$

x	−4	−2	0	2	4
y	0	1	2	3	4



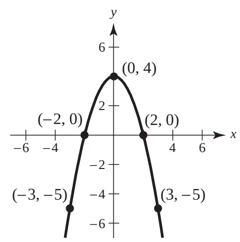
6. $y = 5 - 2x$

x	−1	0	1	2	$\frac{5}{2}$	3	4
y	7	5	3	1	0	−1	−3



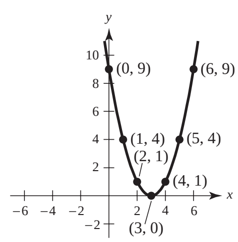
7. $y = 4 - x^2$

x	−3	−2	0	2	3
y	−5	0	4	0	−5



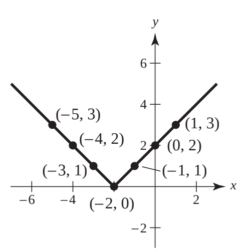
8. $y = (x - 3)^2$

x	0	1	2	3	4	5	6
y	9	4	1	0	1	4	9



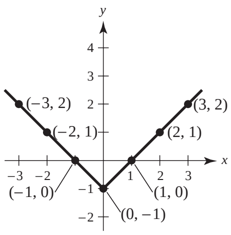
9. $y = |x + 2|$

x	−5	−4	−3	−2	−1	0	1
y	3	2	1	0	1	2	3



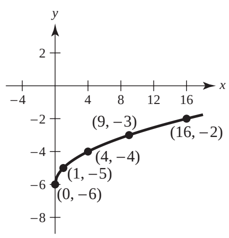
10. $y = |x| - 1$

x	-3	-2	-1	0	1	2	3
y	2	1	0	-1	0	1	2



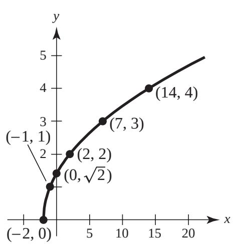
11. $y = \sqrt{x} - 6$

x	0	1	4	9	16
y	-6	-5	-4	-3	-2



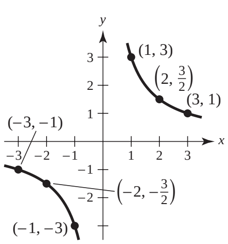
12. $y = \sqrt{x + 2}$

x	-2	-1	0	2	7	14
y	0	1	$\sqrt{2}$	2	3	4



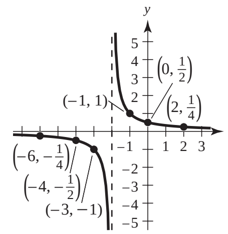
13. $y = \dfrac{3}{x}$

x	-3	-2	-1	0	1	2	3
y	-1	$-\frac{3}{2}$	-3	Undef.	3	$\frac{3}{2}$	1

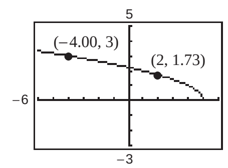


14. $y = \dfrac{1}{x + 2}$

x	-6	-4	-3	-2	-1	0	2
y	$-\frac{1}{4}$	$-\frac{1}{2}$	-1	Undef.	1	$\frac{1}{2}$	$\frac{1}{4}$



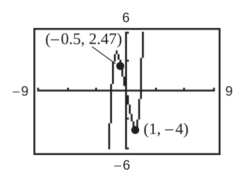
15. $y = \sqrt{5 - x}$



(a) $(2, y) = (2, 1.73)$ $\left(y = \sqrt{5 - 2} = \sqrt{3} \approx 1.73\right)$

(b) $(x, 3) = (-4, 3)$ $\left(3 = \sqrt{5 - (-4)}\right)$

16. $y = x^5 - 5x$



(a) $(-0.5, y) = (-0.5, 2.47)$

(b) $(x, -4) = (-1.65, -4)$ and $(x, -4) = (1, -4)$

17. $y = 2x - 5$

y-intercept: $y = 2(0) - 5 = -5$; $(0, -5)$

x-intercept: $0 = 2x - 5$

$5 = 2x$

$x = \frac{5}{2}$; $\left(\frac{5}{2}, 0\right)$

18. $y = 4x^2 + 3$

y-intercept: $y = 4(0)^2 + 3 = 3$; $(0, 3)$

x-intercept: $0 = 4x^2 + 3$

$-3 = 4x^2$

None. y cannot equal 0.

19. $y = x^2 + x - 2$

y-intercept: $y = 0^2 + 0 - 2$

$y = -2;\ (0, -2)$

x-intercepts: $0 = x^2 + x - 2$

$0 = (x + 2)(x - 1)$

$x = -2, 1;\ (-2, 0), (1, 0)$

20. $y^2 = x^3 - 4x$

y-intercept: $y^2 = 0^3 - 4(0)$

$y = 0;\ (0, 0)$

x-intercepts: $0 = x^3 - 4x$

$0 = x(x - 2)(x + 2)$

$x = 0, \pm 2;\ (0, 0), (\pm 2, 0)$

21. $y = x\sqrt{16 - x^2}$

y-intercept: $y = 0\sqrt{16 - 0^2} = 0;\ (0, 0)$

x-intercepts: $0 = x\sqrt{16 - x^2}$

$0 = x\sqrt{(4 - x)(4 + x)}$

$x = 0, 4, -4;\ (0, 0), (4, 0), (-4, 0)$

22. $y = (x - 1)\sqrt{x^2 + 1}$

y-intercept: $y = (0 - 1)\sqrt{0^2 + 1}$

$y = -1;\ (0, -1)$

x-intercept: $0 = (x - 1)\sqrt{x^2 + 1}$

$x = 1;\ (1, 0)$

23. $y = \dfrac{2 - \sqrt{x}}{5x + 1}$

y-intercept: $y = \dfrac{2 - \sqrt{0}}{5(0) + 1} = 2;\ (0, 2)$

x-intercept: $0 = \dfrac{2 - \sqrt{x}}{5x + 1}$

$0 = 2 - \sqrt{x}$

$x = 4;\ (4, 0)$

24. $y = \dfrac{x^2 + 3x}{(3x + 1)^2}$

y-intercept: $y = \dfrac{0^2 + 3(0)}{[3(0) + 1]^2}$

$y = 0;\ (0, 0)$

x-intercepts: $0 = \dfrac{x^2 + 3x}{(3x + 1)^2}$

$0 = \dfrac{x(x + 3)}{(3x + 1)^2}$

$x = 0, -3;\ (0, 0), (-3, 0)$

25. $x^2y - x^2 + 4y = 0$

y-intercept: $0^2(y) - 0^2 + 4y = 0$

$y = 0;\ (0, 0)$

x-intercept: $x^2(0) - x^2 + 4(0) = 0$

$x = 0;\ (0, 0)$

26. $y = 2x - \sqrt{x^2 + 1}$

y-intercept: $y = 2(0) - \sqrt{0^2 + 1}$

$y = -1;\ (0, -1)$

x-intercept: $0 = 2x - \sqrt{x^2 + 1}$

$2x = \sqrt{x^2 + 1}$

$4x^2 = x^2 + 1$

$3x^2 = 1$

$x^2 = \dfrac{1}{3}$

$x = \pm\dfrac{\sqrt{3}}{3}$

$x = \dfrac{\sqrt{3}}{3};\ \left(\dfrac{\sqrt{3}}{3}, 0\right)$

Note: $x = -\sqrt{3}/3$ is an extraneous solution.

27. Symmetric with respect to the y-axis because $y = (-x)^2 - 6 = x^2 - 6$.

28. $y = x^2 - x$

No symmetry with respect to either axis or the origin.

29. Symmetric with respect to the x-axis because $(-y)^2 = y^2 = x^3 - 8x$.

30. Symmetric with respect to the origin because

$$(-y) = (-x)^3 + (-x)$$
$$-y = -x^3 - x$$
$$y = x^3 + x.$$

31. Symmetric with respect to the origin because $(-x)(-y) = xy = 4.$

32. Symmetric with respect to the x-axis because $x(-y)^2 = xy^2 = -10.$

33. $y = 4 - \sqrt{x + 3}$

No symmetry with respect to either axis or the origin.

34. Symmetric with respect to the origin because

$$(-x)(-y) - \sqrt{4 - (-x)^2} = 0$$
$$xy - \sqrt{4 - x^2} = 0.$$

35. Symmetric with respect to the origin because

$$-y = \frac{-x}{(-x)^2 + 1}$$
$$y = \frac{x}{x^2 + 1}.$$

36. $y = \dfrac{x^2}{x^2 + 1}$ is symmetric with respect to the y-axis because $y = \dfrac{(-x)^2}{(-x)^2 + 1} = \dfrac{x^2}{x^2 + 1}.$

37. $y = |x^3 + x|$ is symmetric with respect to the y-axis because $y = \left|(-x)^3 + (-x)\right| = \left|-(x^3 + x)\right| = \left|x^3 + x\right|.$

38. $|y| - x = 3$ is symmetric with respect to the x-axis because

$$|-y| - x = 3$$
$$|y| - x = 3.$$

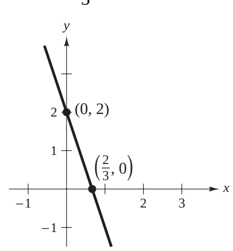
39. $y = 2 - 3x$

$y = 2 - 3(0) = 2,$ y-intercept

$0 = 2 - 3(x) \Rightarrow 3x = 2 \Rightarrow x = \frac{2}{3},$ x-intercept

Intercepts: $(0, 2), \left(\frac{2}{3}, 0\right)$

Symmetry: none



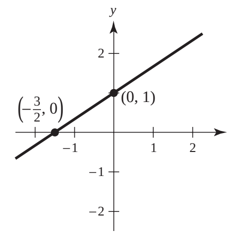
40. $y = \frac{2}{3}x + 1$

$y = \frac{2}{3}(0) + 1 = 1,$ y-intercept

$0 = \frac{2}{3}x + 1 \Rightarrow -\frac{2}{3}x = 1 \Rightarrow x = -\frac{3}{2},$ x-intercept

Intercepts: $(0, 1), \left(-\frac{3}{2}, 0\right)$

Symmetry: none



41. $y = 9 - x^2$

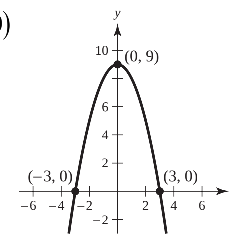
$y = 9 - (0)^2 = 9,$ y-intercept

$0 = 9 - x^2 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3,$ x-intercepts

Intercepts: $(0, 9), (3, 0), (-3, 0)$

$y = 9 - (-x)^2 = 9 - x^2$

Symmetry: y-axis



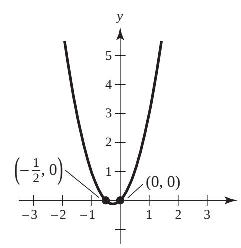
42. $y = 2x^2 + x = x(2x + 1)$

$y = 0(2(0) + 1) = 0,$ y-intercept

$0 = x(2x + 1) \Rightarrow x = 0, -\frac{1}{2},$ x-intercepts

Intercepts: $(0, 0), \left(-\frac{1}{2}, 0\right)$

Symmetry: none



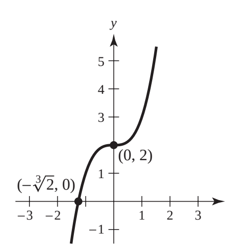
43. $y = x^3 + 2$

$y = 0^3 + 2 = 2,$ y-intercept

$0 = x^3 + 2 \Rightarrow x^3 = -2 \Rightarrow x = -\sqrt[3]{2},$ x-intercept

Intercepts: $\left(-\sqrt[3]{2}, 0\right), (0, 2)$

Symmetry: none



44. $y = x^3 - 4x$

$y = 0^3 - 4(0) = 0$, y-intercept

$x^3 - 4x = 0$

$x(x^2 - 4) = 0$

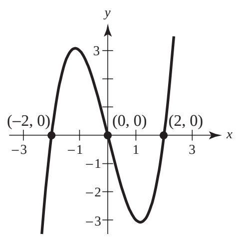
$x(x + 2)(x - 2) = 0$

$x = 0, \pm 2$, x-intercepts

Intercepts: $(0, 0), (2, 0), (-2, 0)$

$y = (-x)^3 - 4(-x) = -x^3 + 4x = -(x^3 - 4x)$

Symmetry: origin



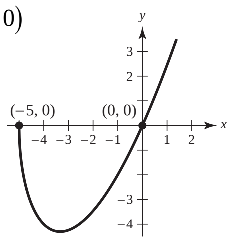
45. $y = x\sqrt{x + 5}$

$y = 0\sqrt{0 + 5} = 0$, y-intercept

$x\sqrt{x + 5} = 0 \Rightarrow x = 0, -5$, x-intercepts

Intercepts: $(0, 0), (-5, 0)$

Symmetry: none



46. $y = \sqrt{25 - x^2}$

$y = \sqrt{25 - 0^2} = \sqrt{25} = 5$, y-intercept

$\sqrt{25 - x^2} = 0$

$25 - x^2 = 0$

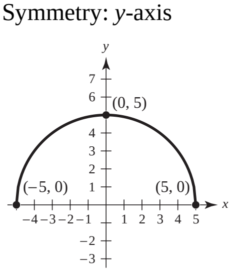
$(5 + x)(5 - x) = 0$

$x = \pm 5$, x-intercept

Intercepts: $(0, 5), (5, 0), (-5, 0)$

$y = \sqrt{25 - (-x)^2} = \sqrt{25 - x^2}$

Symmetry: y-axis



47. $x = y^3$

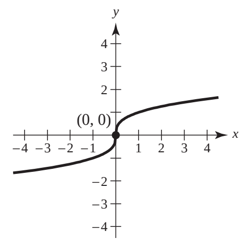
$y^3 = 0 \Rightarrow y = 0$, y-intercept

$x = 0$, x-intercept

Intercept: (0, 0)

$-x = (-y)^3 \Rightarrow -x = -y^3$

Symmetry: origin



48. $x = y^2 - 4$

$y^2 - 4 = 0$

$(y + 2)(y - 2) = 0$

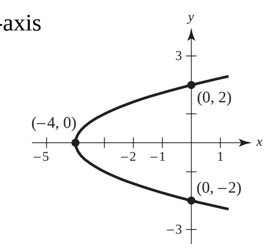
$y = \pm 2$, y-intercepts

$x = 0^2 - 4 = -4$, x-intercept

Intercepts: $(0, 2), (0, -2), (-4, 0)$

$x = (-y)^2 - 4 = y^2 - 4$

Symmetry: x-axis



49. $y = \dfrac{8}{x}$

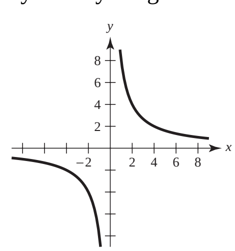
$y = \dfrac{8}{0} \Rightarrow$ Undefined $\Rightarrow$ no y-intercept

$\dfrac{8}{x} = 0 \Rightarrow$ No solution $\Rightarrow$ no x-intercept

Intercepts: none

$-y = \dfrac{8}{-x} \Rightarrow y = \dfrac{8}{x}$

Symmetry: origin



50. $y = \dfrac{10}{x^2 + 1}$

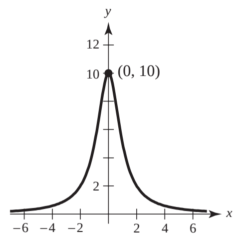
$y = \dfrac{10}{0^2 + 1} = 10$, y-intercept

$\dfrac{10}{x^2 + 1} = 0 \Rightarrow$ No solution $\Rightarrow$ no x-intercepts

Intercept: (0, 10)

$y = \dfrac{10}{(-x)^2 + 1} = \dfrac{10}{x^2 + 1}$

Symmetry: y-axis



51. $y = 6 - |x|$

$y = 6 - |0| = 6$, y-intercept

$6 - |x| = 0$

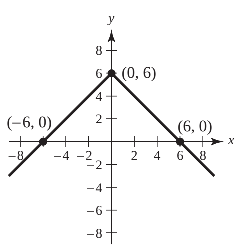
$6 = |x|$

$x = \pm 6$, x-intercepts

Intercepts: $(0, 6), (-6, 0), (6, 0)$

$y = 6 - |-x| = 6 - |x|$

Symmetry: y-axis



52. $y = |6 - x|$

$y = |6 - 0| = |6| = 6$, y-intercept

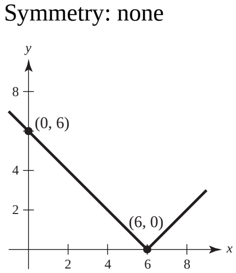
$|6 - x| = 0$

$6 - x = 0$

$6 = x$, x-intercept

Intercepts: (0, 6), (6, 0)

Symmetry: none



53. $y^2 - x = 9$

$y^2 = x + 9$

$y = \pm\sqrt{x + 9}$

$y = \pm\sqrt{0 + 9} = \pm\sqrt{9} = \pm 3$, y-intercepts

$\pm\sqrt{x + 9} = 0$

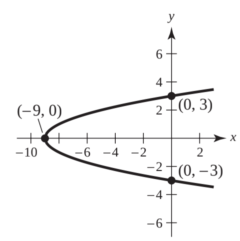
$x + 9 = 0$

$x = -9$, x-intercept

Intercepts: $(0, 3), (0, -3), (-9, 0)$

$(-y)^2 - x = 9 \Rightarrow y^2 - x = 9$

Symmetry: x-axis



54. $x^2 + 4y^2 = 4 \Rightarrow y = \pm\dfrac{\sqrt{4 - x^2}}{2}$

$y = \pm\dfrac{\sqrt{4 - 0^2}}{2} = \pm\dfrac{\sqrt{4}}{2} = \pm 1$, y-intercepts

$x^2 + 4(0)^2 = 4$

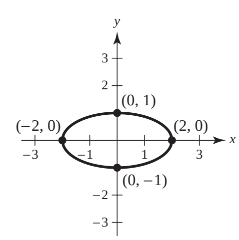
$x^2 = 4$

$x = \pm 2$, x-intercepts

Intercepts: $(-2, 0), (2, 0), (0, -1), (0, 1)$

$(-x)^2 + 4(-y)^2 = 4 \Rightarrow x^2 + 4y^2 = 4$

Symmetry: origin and both axes



55. $x + 3y^2 = 6$

$$3y^2 = 6 - x$$

$$y = \pm\sqrt{\frac{6-x}{3}}$$

$$y = \pm\sqrt{\frac{6-0}{3}} = \pm\sqrt{2},\ y\text{-intercepts}$$

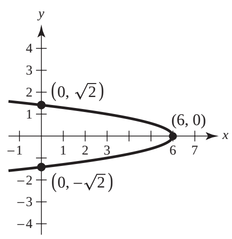
$$x + 3(0)^2 = 6$$

$$x = 6,\ x\text{-intercept}$$

Intercepts: $(6, 0), \left(0, \sqrt{2}\right), \left(0, -\sqrt{2}\right)$

$x + 3(-y)^2 = 6 \Rightarrow x + 3y^2 = 6$

Symmetry: x-axis



56. $3x - 4y^2 = 8$

$$4y^2 = 3x - 8$$

$$y = \pm\sqrt{\tfrac{3}{4}x - 2}$$

$$y = \pm\sqrt{\frac{3}{4}(0) - 2} = \pm\sqrt{-2}$$

$\Rightarrow$ no solution $\Rightarrow$ no y-intercepts

$$3x - 4(0)^2 = 8$$

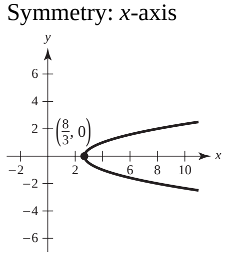
$$3x = 8$$

$$x = \frac{8}{3},\ x\text{-intercept}$$

Intercept: $\left(\frac{8}{3}, 0\right)$

$3x - 4(-y)^2 = 8 \Rightarrow 3x - 4y^2 = 8$

Symmetry: x-axis



57. $x + y = 8 \Rightarrow y = 8 - x$

$4x - y = 7 \Rightarrow y = 4x - 7$

$$8 - x = 4x - 7$$

$$15 = 5x$$

$$3 = x$$

The corresponding y-value is $y = 5$.

Point of intersection: $(3, 5)$

58. $3x - 2y = -4 \Rightarrow y = \dfrac{3x + 4}{2}$

$4x + 2y = -10 \Rightarrow y = \dfrac{-4x - 10}{2}$

$$\frac{3x+4}{2} = \frac{-4x-10}{2}$$

$$3x + 4 = -4x - 10$$

$$7x = -14$$

$$x = -2$$

The corresponding y-value is $y = -1$.

Point of intersection: $(-2, -1)$

59. $x^2 + y = 6 \Rightarrow y = 6 - x^2$

$x + y = 4 \Rightarrow y = 4 - x$

$$6 - x^2 = 4 - x$$

$$0 = x^2 - x - 2$$

$$0 = (x-2)(x+1)$$

$$x = 2, -1$$

The corresponding y-values are $y = 2$ (for $x = 2$) and $y = 5$ (for $x = -1$).

Points of intersection: $(2, 2), (-1, 5)$

60. $x = 3 - y^2 \Rightarrow y^2 = 3 - x$

$y = x - 1$

$$3 - x = (x-1)^2$$

$$3 - x = x^2 - 2x + 1$$

$$0 = x^2 - x - 2 = (x+1)(x-2)$$

$$x = -1 \text{ or } x = 2$$

The corresponding y-values are $y = -2$ (for $x = -1$) and $y = 1$ (for $x = 2$).

Points of intersection: $(-1, -2), (2, 1)$

61. $x^2 + y^2 = 5 \Rightarrow y^2 = 5 - x^2$

$x - y = 1 \Rightarrow y = x - 1$

$5 - x^2 = (x - 1)^2$

$5 - x^2 = x^2 - 2x + 1$

$0 = 2x^2 - 2x - 4 = 2(x + 1)(x - 2)$

$x = -1$ or $x = 2$

The corresponding y-values are $y = -2$ (for $x = -1$) and $y = 1$ (for $x = 2$).

Points of intersection: $(-1, -2), (2, 1)$

62. $x^2 + y^2 = 25 \Rightarrow y^2 = 25 - x^2$

$-3x + y = 15 \Rightarrow y = 3x + 15$

$25 - x^2 = (3x + 15)^2$

$25 - x^2 = 9x^2 + 90x + 225$

$0 = 10x^2 + 90x + 200$

$0 = x^2 + 9x + 20$

$0 = (x + 5)(x + 4)$

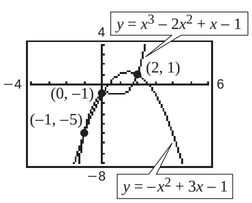
$x = -4$ or $x = -5$

The corresponding y-values are $y = 3$ (for $x = -4$) and $y = 0$ (for $x = -5$).

Points of intersection: $(-4, 3), (-5, 0)$

63. $y = x^3 - 2x^2 + x - 1$

$y = -x^2 + 3x - 1$



Points of intersection: $(-1, -5), (0, -1), (2, 1)$

Analytically, $x^3 - 2x^2 + x - 1 = -x^2 + 3x - 1$

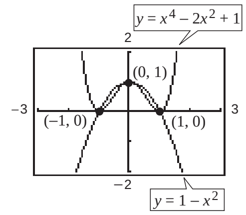
$x^3 - x^2 - 2x = 0$

$x(x - 2)(x + 1) = 0$

$x = -1, 0, 2.$

64. $y = x^4 - 2x^2 + 1$

$y = 1 - x^2$



Points of intersection: $(-1, 0), (0, 1), (1, 0)$

Analytically, $1 - x^2 = x^4 - 2x^2 + 1$

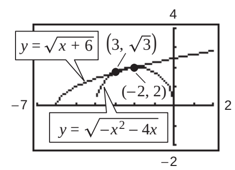
$0 = x^4 - x^2$

$0 = x^2(x + 1)(x - 1)$

$x = -1, 0, 1.$

65. $y = \sqrt{x + 6}$

$y = \sqrt{-x^2 - 4x}$



Points of intersection: $(-2, 2), \left(-3, \sqrt{3}\right) \approx (-3, 1.732)$

Analytically, $\sqrt{x + 6} = \sqrt{-x^2 - 4x}$

$x + 6 = -x^2 - 4x$

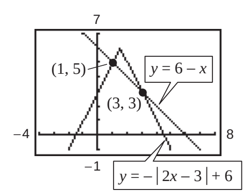
$x^2 + 5x + 6 = 0$

$(x + 3)(x + 2) = 0$

$x = -3, -2.$

66. $y = -|2x - 3| + 6$

$y = 6 - x$



Points of intersection: (3, 3), (1, 5)

Analytically, $-|2x - 3| + 6 = 6 - x$

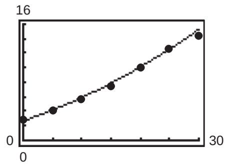
$|2x - 3| = x$

$2x - 3 = x$ or $2x - 3 = -x$

$x = 3$ or $x = 1.$

67. (a) Using a graphing utility, you obtain $y = 0.005t^2 + 0.27t + 2.7.$

(b)



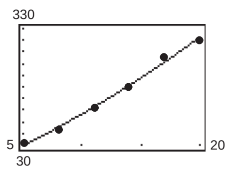
(c) For 2020, $t = 40.$

$$y = 0.005(40)^2 + 0.27(40) + 2.7$$
$$= 21.5$$

The GDP in 2020 will be \$21.5 trillion.

68. (a) Using a graphing utility, you obtain $y = 0.24t^2 + 12.6t - 40.$

(b)



The model is a good fit for the data.

(c) For 2020, $t = 30.$

$$y = 0.24(30)^2 + 12.6(30) - 40$$
$$= 554$$

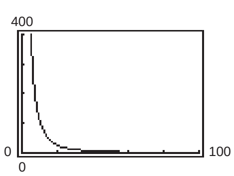
The number of cellular phone subscribers in 2020 will be 554 million.

69.

$$C = R$$
$$2.04x + 5600 = 3.29x$$
$$5600 = 3.29x - 2.04x$$
$$5600 = 1.25x$$
$$x = \frac{5600}{1.25} = 4480$$

To break even, 4480 units must be sold.

70. $y = \dfrac{10{,}770}{x^2} - 0.37$



If the diameter is doubled, the resistance is changed by approximately a factor of $\frac{1}{4}$. For instance, $y(20) \approx 26.555$ and $y(40) \approx 6.36125$.

71. $y = kx^3$

(a) $(1, 4)$: $\quad 4 = k(1)^3 \Rightarrow k = 4$

(b) $(-2, 1)$: $\quad 1 = k(-2)^3 = -8k \Rightarrow k = -\frac{1}{8}$

(c) $(0, 0)$: $\quad 0 = k(0)^3 \Rightarrow k$ can be any real number.

(d) $(-1, -1)$: $\quad -1 = k(-1)^3 = -k \Rightarrow k = 1$

72. $y^2 = 4kx$

(a) $(1, 1)$:

$$1^2 = 4k(1)$$
$$1 = 4k$$
$$k = \frac{1}{4}$$

(b) $(2, 4)$:

$$(4)^2 = 4k(2)$$
$$16 = 8k$$
$$k = 2$$

(c) $(0, 0)$: $\quad 0^2 = 4k(0)$

k can be any real number.

(d) $(3, 3)$:

$$(3)^2 = 4k(3)$$
$$9 = 12k$$
$$k = \frac{9}{12} = \frac{3}{4}$$

73. Answers may vary. *Sample answer*:

$y = (x + 4)(x - 3)(x - 8)$ has intercepts at $x = -4$, $x = 3$, and $x = 8$.

74. Answers may vary. *Sample answer*:

$y = \left(x + \frac{3}{2}\right)(x - 4)\left(x - \frac{5}{2}\right)$ has intercepts at $x = -\frac{3}{2}$, $x = 4$, and $x = \frac{5}{2}$.

75. (a) If (x, y) is on the graph, then so is $(-x, y)$ by y-axis symmetry. Because $(-x, y)$ is on the graph, then so is $(-x, -y)$ by x-axis symmetry. So, the graph is symmetric with respect to the origin. The converse is not true. For example, $y = x^3$ has origin symmetry but is not symmetric with respect to either the x-axis or the y-axis.

(b) Assume that the graph has x-axis and origin symmetry. If (x, y) is on the graph, so is $(x, -y)$ by x-axis symmetry. Because $(x, -y)$ is on the graph, then so is $(-x, -(-y)) = (-x, y)$ by origin symmetry. Therefore, the graph is symmetric with respect to the y-axis. The argument is similar for y-axis and origin symmetry.

76. (a) Intercepts for $y = x^3 - x$:

y-intercept: $y = 0^3 - 0 = 0$; $(0, 0)$

x-intercepts: $0 = x^3 - x = x(x^2 - 1) = x(x - 1)(x + 1)$;

$(0, 0), (1, 0)\ (-1, 0)$

Intercepts for $y = x^2 + 2$:

y-intercept: $y = 0 + 2 = 2$; $(0, 2)$

x-intercepts: $0 = x^2 + 2$

None. y cannot equal 0.

(b) Symmetry with respect to the origin for $y = x^3 - x$ because $-y = (-x)^3 - (-x) = -x^3 + x$.

Symmetry with respect to the y-axis for $y = x^2 + 2$ because $y = (-x)^2 + 2 = x^2 + 2$.

(c)
$$x^3 - x = x^2 + 2$$
$$x^3 - x^2 - x - 2 = 0$$
$$(x - 2)(x^2 + x + 1) = 0$$
$$x = 2 \Rightarrow y = 6$$

Point of intersection : (2, 6)

Note: The polynomial $x^2 + x + 1$ has no real roots.

77. False. x-axis symmetry means that if $(-4, -5)$ is on the graph, then $(-4, 5)$ is also on the graph. For example, $(4, -5)$ is not on the graph of $x = y^2 - 29$, whereas $(-4, -5)$ is on the graph.

78. True. $f(4) = f(-4)$.

79. True. The x-intercepts are $\left(\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}, 0\right)$.

80. True. The x-intercept is $\left(-\dfrac{b}{2a}, 0\right)$.

Section 1.2 Linear Models and Rates of Change

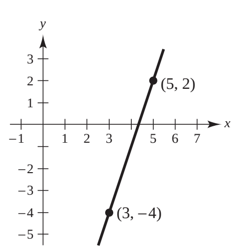
1. $m = 2$

2. $m = 0$

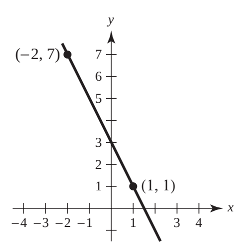
3. $m = -1$

4. $m = -12$

5. $m = \dfrac{2 - (-4)}{5 - 3} = \dfrac{6}{2} = 3$

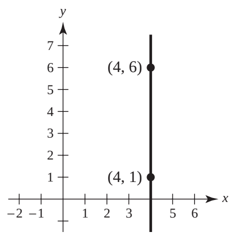


6. $m = \dfrac{7 - 1}{-2 - 1} = \dfrac{6}{-3} = -2$



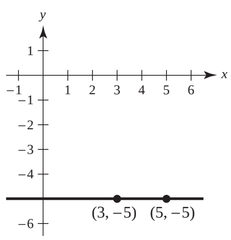
7. $m = \dfrac{1 - 6}{4 - 4} = \dfrac{-5}{0}$, undefined.

The line is vertical.

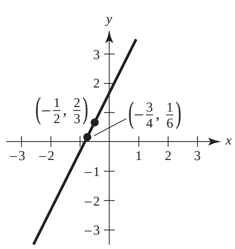


8. $m = \dfrac{-5 - (-5)}{5 - 3} = \dfrac{0}{2} = 0$

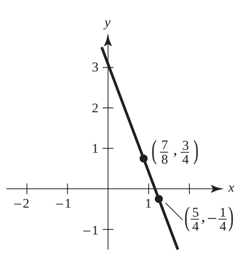
The line is horizontal.



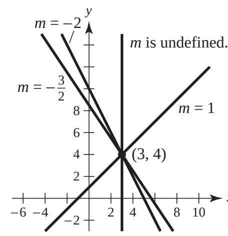
9. $m = \dfrac{\frac{2}{3} - \frac{1}{6}}{-\frac{1}{2} - \left(-\frac{3}{4}\right)} = \dfrac{\frac{1}{2}}{\frac{1}{4}} = 2$



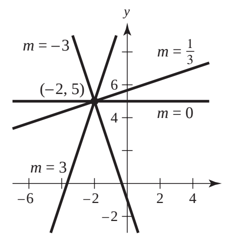
10. $m = \dfrac{\left(\frac{3}{4}\right) - \left(-\frac{1}{4}\right)}{\left(\frac{7}{8}\right) - \left(\frac{5}{4}\right)} = \dfrac{1}{-\frac{3}{8}} = -\dfrac{8}{3}$



11.



12.



13. Because the slope is 0, the line is horizontal and its equation is $y = 2$. Therefore, three additional points are $(0, 2)$, $(1, 2)$, $(5, 2)$.

14. Because the slope is undefined, the line is vertical and its equation is $x = -4$. Therefore, three additional points are $(-4, 0)$, $(-4, 1)$, $(-4, 2)$.

15. The equation of this line is

$$y - 7 = -3(x - 1)$$
$$y = -3x + 10.$$

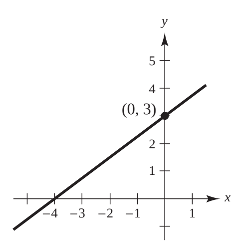
Therefore, three additional points are (0, 10), (2, 4), and (3, 1).

16. The equation of this line is

$$y + 2 = 2(x + 2)$$
$$y = 2x + 2.$$

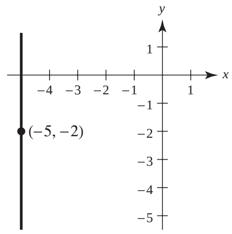
Therefore, three additional points are $(-3, -4)$, $(-1, 0)$, and (0, 2).

17.
$$y = \tfrac{3}{4}x + 3$$
$$4y = 3x + 12$$
$$0 = 3x - 4y + 12$$

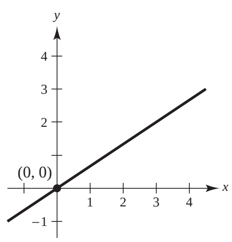


18. The slope is undefined so the line is vertical.

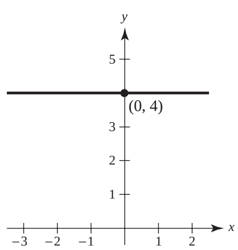
$$x = -5$$
$$x + 5 = 0$$



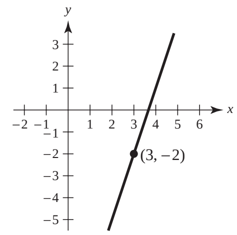
19.
$$y = \tfrac{2}{3}x$$
$$3y = 2x$$
$$0 = 2x - 3y$$



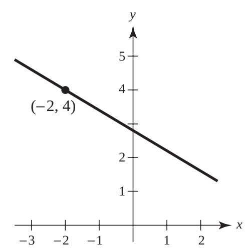
20.
$$y = 4$$
$$y - 4 = 0$$



21.
$$y + 2 = 3(x - 3)$$
$$y + 2 = 3x - 9$$
$$y = 3x - 11$$
$$0 = 3x - y - 11$$

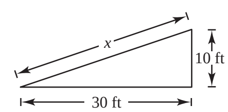


22.
$$y - 4 = -\tfrac{3}{5}(x + 2)$$
$$5y - 20 = -3x - 6$$
$$3x + 5y - 14 = 0$$



23. (a) Slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{1}{3}$

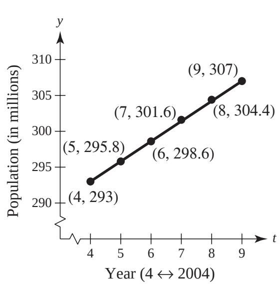
(b)



By the Pythagorean Theorem,

$$x^2 = 30^2 + 10^2 = 1000$$
$$x = 10\sqrt{10} \approx 31.623 \text{ feet.}$$

24. (a)



(b) The slopes are:
$$\frac{295.8 - 293.0}{5 - 4} = 2.8$$
$$\frac{298.6 - 295.8}{6 - 5} = 2.8$$
$$\frac{301.6 - 298.6}{7 - 6} = 3.0$$
$$\frac{304.4 - 301.6}{8 - 7} = 2.8$$
$$\frac{307.0 - 304.4}{9 - 8} = 2.6$$

The population increased least rapidly from 2008 to 2009.

(c) Average rate of change from 2004 to 2009:

$$\frac{307.0 - 293.0}{9 - 4} = \frac{14}{5}$$
$$= 2.8 \text{ million per yr}$$

(d) For 2020, $t = 20$ and $y \approx 16(2.8) + 293.0 = 337.8$ million.

[Equivalently, $y \approx 11(2.8) + 307.0 = 337.8$.]

25. $y = 4x - 3$

The slope is $m = 4$ and the y-intercept is $(0, -3)$.

26. $-x + y = 1$

$$y = x + 1$$

The slope is $m = 1$ and the y-intercept is (0, 1).

27. $x + 5y = 20$

$$y = -\tfrac{1}{5}x + 4$$

Therefore, the slope is $m = -\frac{1}{5}$ and the y-intercept is (0, 4).

28. $6x - 5y = 15$

$$y = \tfrac{6}{5}x - 3$$

Therefore, the slope is $m = \frac{6}{5}$ and the y-intercept is $(0, -3)$.

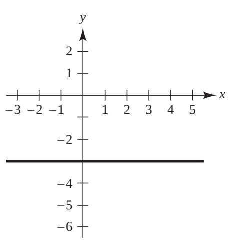
29. $x = 4$

The line is vertical. Therefore, the slope is undefined and there is no y-intercept.

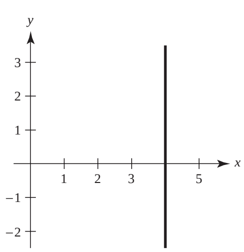
30. $y = -1$

The line is horizontal. Therefore, the slope is $m = 0$ and the y-intercept is $(0, -1)$.

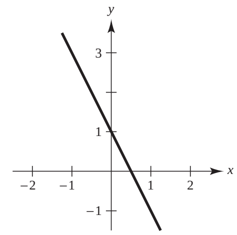
31. $y = -3$



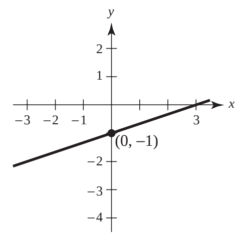
32. $x = 4$



33. $y = -2x + 1$

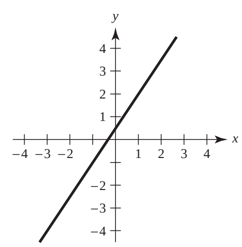


34. $y = \frac{1}{3}x - 1$



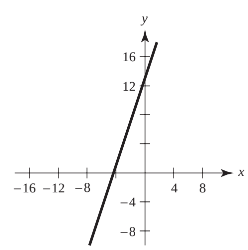
35. $y - 2 = \frac{3}{2}(x - 1)$

$$y = \tfrac{3}{2}x + \tfrac{1}{2}$$



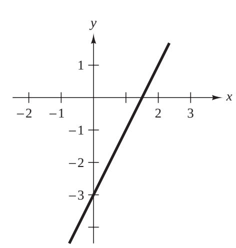
36. $y - 1 = 3(x + 4)$

$$y = 3x + 13$$



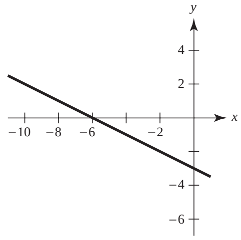
37. $2x - y - 3 = 0$

$$y = 2x - 3$$



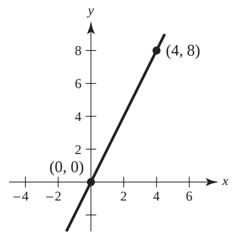
38. $x + 2y + 6 = 0$

$$y = -\tfrac{1}{2}x - 3$$



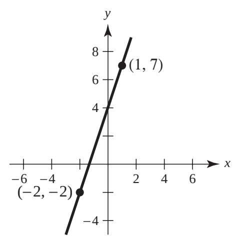
39. $m = \dfrac{8 - 0}{4 - 0} = 2$

$$y - 0 = 2(x - 0)$$
$$y = 2x$$
$$0 = 2x - y$$



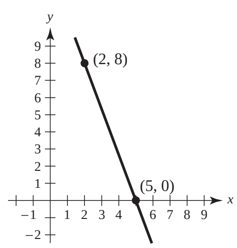
40. $m = \dfrac{7 - (-2)}{1 - (-2)} = \dfrac{9}{3} = 3$

$$y - (-2) = 3(x - (-2))$$
$$y + 2 = 3(x + 2)$$
$$y = 3x + 4$$
$$0 = 3x - y + 4$$



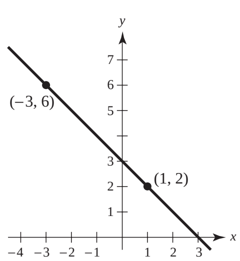
41. $m = \dfrac{8 - 0}{2 - 5} = -\dfrac{8}{3}$

$$y - 0 = -\frac{8}{3}(x - 5)$$
$$y = -\frac{8}{3}x + \frac{40}{3}$$
$$8x + 3y - 40 = 0$$



42. $m = \dfrac{6 - 2}{-3 - 1} = \dfrac{4}{-4} = -1$

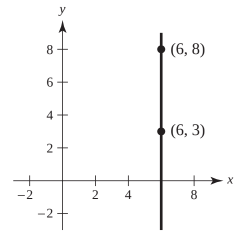
$$y - 2 = -1(x - 1)$$
$$y - 2 = -x + 1$$
$$x + y - 3 = 0$$



43. $m = \dfrac{8 - 3}{6 - 6} = \dfrac{5}{0}$, undefined

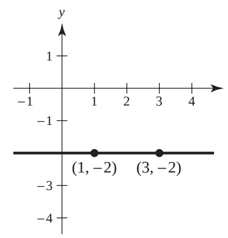
The line is horizontal.

$$x = 6$$
$$x - 6 = 0$$



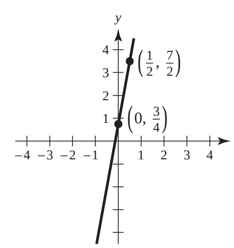
44. $m = \dfrac{-2 - (-2)}{3 - 1} = \dfrac{0}{2} = 0$

$$y = -2$$
$$y + 2 = 0$$



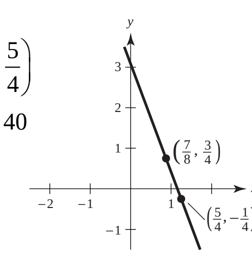
45. $m = \dfrac{\frac{7}{2} - \frac{3}{4}}{\frac{1}{2} - 0} = \dfrac{\frac{11}{4}}{\frac{1}{2}} = \dfrac{11}{2}$

$$y - \frac{3}{4} = \frac{11}{2}(x - 0)$$
$$y = \frac{11}{2}x + \frac{3}{4}$$
$$0 = 22x - 4y + 3$$



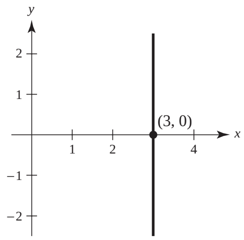
46. $m = \dfrac{\left(\frac{3}{4}\right) - \left(-\frac{1}{4}\right)}{\left(\frac{7}{8}\right) - \left(\frac{5}{4}\right)} = \dfrac{1}{-\frac{3}{8}} = -\dfrac{8}{3}$

$$y + \frac{1}{4} = \frac{-8}{3}\left(x - \frac{5}{4}\right)$$
$$12y + 3 = -32x + 40$$
$$32x + 12y - 37 = 0$$



47. $x = 3$

$x - 3 = 0$

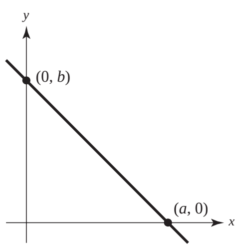


48. $m = -\dfrac{b}{a}$

$$y = \frac{-b}{a}x + b$$

$$\frac{b}{a}x + y = b$$

$$\frac{x}{a} + \frac{y}{b} = 1$$



49. $\dfrac{x}{2} + \dfrac{y}{3} = 1$

$3x + 2y - 6 = 0$

50. $\dfrac{x}{-\frac{2}{3}} + \dfrac{y}{-2} = 1$

$$\frac{-3x}{2} - \frac{y}{2} = 1$$

$$3x + y = -2$$

$$3x + y + 2 = 0$$

51. $\dfrac{x}{a} + \dfrac{y}{a} = 1$

$$\frac{1}{a} + \frac{2}{a} = 1$$

$$\frac{3}{a} = 1$$

$$a = 3 \Rightarrow x + y = 3$$

$$x + y - 3 = 0$$

52. $\dfrac{x}{a} + \dfrac{y}{a} = 1$

$$\frac{-3}{a} + \frac{4}{a} = 1$$

$$\frac{1}{a} = 1$$

$$a = 1 \Rightarrow x + y = 1$$

$$x + y - 1 = 0$$

53. $\dfrac{x}{2a} + \dfrac{y}{a} = 1$

$$\frac{9}{2a} + \frac{-2}{a} = 1$$

$$\frac{9 - 4}{2a} = 1$$

$$5 = 2a$$

$$a = \frac{5}{2}$$

$$\frac{x}{2\left(\frac{5}{2}\right)} + \frac{y}{\left(\frac{5}{2}\right)} = 1$$

$$\frac{x}{5} + \frac{2y}{5} = 1$$

$$x + 2y = 5$$

$$x + 2y - 5 = 0$$

54. $\dfrac{x}{a} + \dfrac{y}{-a} = 1$

$$\frac{\left(-\frac{2}{3}\right)}{a} + \frac{(-2)}{-a} = 1$$

$$-\frac{2}{3} + 2 = a$$

$$a = \frac{4}{3}$$

$$\frac{x}{\left(\frac{4}{3}\right)} + \frac{y}{\left(-\frac{4}{3}\right)} = 1$$

$$x - y = \frac{4}{3}$$

$$3x - 3y - 4 = 0$$

55. The given line is vertical.

(a) $x = -7$, or $x + 7 = 0$

(b) $y = -2$, or $y + 2 = 0$

56. The given line is horizontal.

(a) $y = 0$

(b) $x = -1$, or $x + 1 = 0$

57. $x - y = -2$

$y = x + 2$

$m = 1$

(a) $y - 5 = 1(x - 2)$

$y - 5 = x - 2$

$x - y + 3 = 0$

(b) $y - 5 = -1(x - 2)$

$y - 5 = -x + 2$

$x + y - 7 = 0$

58. $x + y = 7$

$y = -x + 7$

$m = -1$

(a) $y - 2 = -1(x + 3)$

$y - 2 = -x - 3$

$x + y + 1 = 0$

(b) $y - 2 = 1(x + 3)$

$y - 2 = x + 3$

$0 = x - y + 5$

59. $4x - 2y = 3$

$y = 2x - \frac{3}{2}$

$m = 2$

(a) $y - 1 = 2(x - 2)$

$y - 1 = 2x - 4$

$0 = 2x - y - 3$

(b) $y - 1 = -\frac{1}{2}(x - 2)$

$2y - 2 = -x + 2$

$x + 2y - 4 = 0$

60. $7x + 4y = 8$

$4y = -7x + 8$

$y = \frac{-7}{4}x + 2$

$m = -\frac{7}{4}$

(a) $y + \frac{1}{2} = \frac{-7}{4}\left(x - \frac{5}{6}\right)$

$y + \frac{1}{2} = \frac{-7}{4}x + \frac{35}{24}$

$24y + 12 = -42x + 35$

$42x + 24y - 23 = 0$

(b) $y + \frac{1}{2} = \frac{4}{7}\left(x - \frac{5}{6}\right)$

$42y + 21 = 24x - 20$

$24x - 42y - 41 = 0$

61. $5x - 3y = 0$

$y = \frac{5}{3}x$

$m = \frac{5}{3}$

(a) $y - \frac{7}{8} = \frac{5}{3}\left(x - \frac{3}{4}\right)$

$24y - 21 = 40x - 30$

$0 = 40x - 24y - 9$

(b) $y - \frac{7}{8} = -\frac{3}{5}\left(x - \frac{3}{4}\right)$

$40y - 35 = -24x + 18$

$24x + 40y - 53 = 0$

62. $3x + 4y = 7$

$4y = -3x + 7$

$y = -\frac{3}{4}x + \frac{7}{4}$

$m = -\frac{3}{4}$

(a) $y - (-5) = -\frac{3}{4}(x - 4)$

$y + 5 = -\frac{3}{4}x + 3$

$4y + 20 = -3x + 12$

$3x + 4y + 8 = 0$

(b) $y - (-5) = \frac{4}{3}(x - 4)$

$y + 5 = \frac{4}{3}x - \frac{16}{3}$

$3y + 15 = 4x - 16$

$0 = 4x - 3y - 31$

63. The slope is 250.

$V = 1850$ when $t = 2$.

$V = 250(t - 2) + 1850$

$= 250t + 1350$

64. The slope is 4.50.

$V = 156$ when $t = 2$.

$V = 4.5(t - 2) + 156$

$= 4.5t + 147$

65. The slope is -1600.

$V = 17{,}200$ when $t = 2$.

$V = -1600(t - 2) + 17{,}200$

$= -1600t + 20{,}400$

66. The slope is -5600.

$V = 245{,}000$ when $t = 2$.

$V = -5600(t - 2) + 245{,}000$

$= -5600t + 256{,}200$

67. $m_1 = \dfrac{1 - 0}{-2 - (-1)} = -1$

$m_2 = \dfrac{-2 - 0}{2 - (-1)} = -\dfrac{2}{3}$

$m_1 \neq m_2$

The points are not collinear.

68. $m_1 = \dfrac{-6 - 4}{7 - 0} = -\dfrac{10}{7}$

$m_2 = \dfrac{11 - 4}{-5 - 0} = -\dfrac{7}{5}$

$m_1 \neq m_2$

The points are not collinear.

69. Equations of perpendicular bisectors:

$y - \dfrac{c}{2} = \dfrac{a - b}{c}\left(x - \dfrac{a + b}{2}\right)$

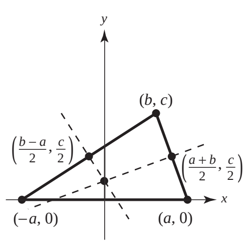
$y - \dfrac{c}{2} = \dfrac{a + b}{-c}\left(x - \dfrac{b - a}{2}\right)$

Setting the right-hand sides of the two equations equal and solving for x yields $x = 0$.

Letting $x = 0$ in either equation gives the point of intersection:

$\left(0, \dfrac{-a^2 + b^2 + c^2}{2c}\right).$

This point lies on the third perpendicular bisector, $x = 0$.

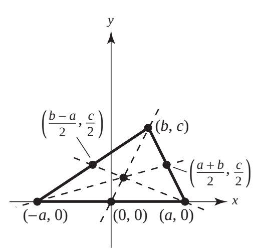


70. Equations of medians:

$y = \dfrac{c}{b}x$

$y = \dfrac{c}{3a + b}(x + a)$

$y = \dfrac{c}{-3a + b}(x - a)$



Solving simultaneously, the point of intersection is $\left(\dfrac{b}{3}, \dfrac{c}{3}\right)$.

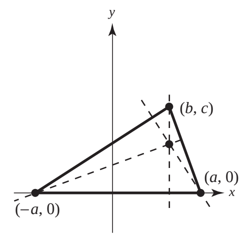
71. Equations of altitudes:

$y = \dfrac{a - b}{c}(x + a)$

$x = b$

$y = -\dfrac{a + b}{c}(x - a)$

Solving simultaneously, the point of intersection is $\left(b, \dfrac{a^2 - b^2}{c}\right)$.



72. The slope of the line segment from $\left(\dfrac{b}{3}, \dfrac{c}{3}\right)$ to $\left(b, \dfrac{a^2 - b^2}{c}\right)$ is:

$m_1 = \dfrac{\left[(a^2 - b^2)/c\right] - (c/3)}{b - (b/3)}$

$= \dfrac{(3a^2 - 3b^2 - c^2)/(3c)}{(2b)/3} = \dfrac{3a^2 - 3b^2 - c^2}{2bc}$

The slope of the line segment from $\left(\dfrac{b}{3}, \dfrac{c}{3}\right)$ to $\left(0, \dfrac{-a^2 + b^2 + c^2}{2c}\right)$ is:

$m_2 = \dfrac{\left[(-a^2 + b^2 + c^2)/(2c)\right] - (c/3)}{0 - (b/3)}$

$= \dfrac{(-3a^2 + 3b^2 + 3c^2 - 2c^2)/(6c)}{-b/3} = \dfrac{3a^2 - 3b^2 - c^2}{2bc}$

$m_1 = m_2$

Therefore, the points are collinear.

73. $ax + by = 4$

(a) The line is parallel to the x-axis if $a = 0$ and $b \neq 0$.

(b) The line is parallel to the y-axis if $b = 0$ and $a \neq 0$.

(c) Answers will vary. *Sample answer*: $a = -5$ and $b = 8$.

$$-5x + 8y = 4$$
$$y = \frac{1}{8}(5x + 4) = \frac{5}{8}x + \frac{1}{2}$$

(d) The slope must be $-\frac{5}{2}$.

Answers will vary. *Sample answer*: $a = 5$ and $b = 2$.

$$5x + 2y = 4$$
$$y = \frac{1}{2}(-5x + 4) = -\frac{5}{2}x + 2$$

(e) $a = \frac{5}{2}$ and $b = 3$.

$$\frac{5}{2}x + 3y = 4$$
$$5x + 6y = 8$$

74. (a) Lines c, d, e and f have positive slopes.

(b) Lines a and b have negative slopes.

(c) Lines c and e appear parallel.
Lines d and f appear parallel.

(d) Lines b and f appear perpendicular.
Lines b and d appear perpendicular.

75. Find the equation of the line through the points (0, 32) and (100, 212).

$$m = \frac{180}{100} = \frac{9}{5}$$
$$F - 32 = \frac{9}{5}(C - 0)$$
$$F = \frac{9}{5}C + 32$$

or

$$C = \frac{1}{9}(5F - 160)$$
$$5F - 9C - 160 = 0$$

For $F = 72°$, $C \approx 22.2°$.

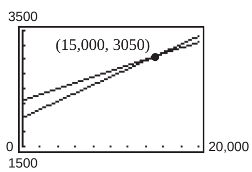
76. $C = 0.51x + 200$

For $x = 137$, $C = 0.51(137) + 200 = \$269.87$.

77. (a) Current job: $W_1 = 0.07s + 2000$

New job offer: $W_2 = 0.05s + 2300$

(b)



Using a graphing utility, the point of intersection is (15,000, 3050).

Analytically, $W_1 = W_2$

$$0.07s + 2000 = 0.05s + 2300$$
$$0.02s = 300$$
$$s = 15{,}000$$

So, $W_1 = W_2 = 0.07(15{,}000) + 2000 = 3050$.

When sales exceed \$15,000, the current job pays more.

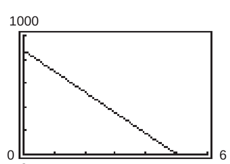
(c) No, if you can sell \$20,000 worth of goods, then $W_1 > W_2$.

(**Note:** $W_1 = 3400$ and $W_2 = 3300$ when $s = 20{,}000$.)

78. (a) Depreciation per year:

$$\frac{875}{5} = \$175$$
$$y = 875 - 175x$$

where $0 \leq x \leq 5$.



(b) $y = 875 - 175(2) = \$525$

(c)
$$200 = 875 - 175x$$
$$175x = 675$$
$$x \approx 3.86 \text{ years}$$

79. (a) Two points are (50, 780) and (47, 825). The slope is

$$m = \frac{825 - 780}{47 - 50} = \frac{45}{-3} = -15.$$

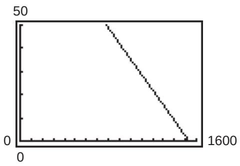
$$p - 780 = -15(x - 50)$$

$$p = -15x + 750 + 780 = -15x + 1530$$

or

$$x = \frac{1}{15}(1530 - p)$$

(b)



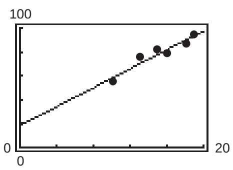
If $p = 855$, then $x = 45$ units.

(c) If $p = 795$, then $x = \frac{1}{15}(1530 - 795) = 49$ units

80. (a) $y = 18.91 + 3.97x$

$(x = \text{quiz score},\ y = \text{test score})$

(b)

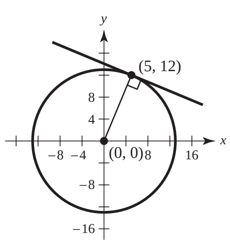


(c) If $x = 17$, $y = 18.91 + 3.97(17) = 86.4$.

(d) The slope shows the average increase in exam score for each unit increase in quiz score.

(e) The points would shift vertically upward 4 units. The new regression line would have a y-intercept 4 greater than before: $y = 22.91 + 3.97x$.

81. The tangent line is perpendicular to the line joining the point (5, 12) and the center (0, 0).



Slope of the line joining (5, 12) and (0, 0) is $\frac{12}{5}$.

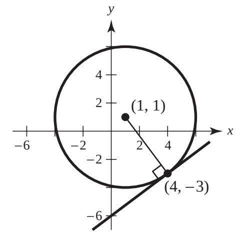
The equation of the tangent line is

$$y - 12 = \frac{-5}{12}(x - 5)$$

$$y = \frac{-5}{12}x + \frac{169}{12}$$

$$5x + 12y - 169 = 0.$$

82. The tangent line is perpendicular to the line joining the point $(4, -3)$ and the center of the circle, (1, 1).



Slope of the line joining (1, 1) and $(4, -3)$ is

$$\frac{1 + 3}{1 - 4} = \frac{-4}{3}.$$

Tangent line:

$$y + 3 = \frac{3}{4}(x - 4)$$

$$y = \frac{3}{4}x - 6$$

$$0 = 3x - 4y - 24$$

83. $x - y - 2 = 0 \Rightarrow d = \dfrac{\left|1(-2) + (-1)(1) - 2\right|}{\sqrt{1^2 + 1^2}} = \dfrac{5}{\sqrt{2}} = \dfrac{5\sqrt{2}}{2}$

84. $4x + 3y - 10 = 0 \Rightarrow d = \dfrac{\left|4(2) + 3(3) - 10\right|}{\sqrt{4^2 + 3^2}} = \dfrac{7}{5}$

85. A point on the line $x + y = 1$ is (0, 1). The distance from the point (0, 1) to $x + y - 5 = 0$ is

$$d = \frac{\left|1(0) + 1(1) - 5\right|}{\sqrt{1^2 + 1^2}} = \frac{\left|1 - 5\right|}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}.$$

86. A point on the line $3x - 4y = 1$ is $(-1, -1)$. The distance from the point $(-1, -1)$ to $3x - 4y - 10 = 0$ is

$$d = \frac{\left|3(-1) - 4(-1) - 10\right|}{\sqrt{3^2 + (-4)^2}} = \frac{\left|-3 + 4 - 10\right|}{5} = \frac{9}{5}.$$

87. If $A = 0$, then $By + C = 0$ is the horizontal line $y = -C/B$. The distance to (x_1, y_1) is

$$d = \left|y_1 - \left(\frac{-C}{B}\right)\right| = \frac{|By_1 + C|}{|B|} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

If $B = 0$, then $Ax + C = 0$ is the vertical line $x = -C/A$. The distance to (x_1, y_1) is

$$d = \left|x_1 - \left(\frac{-C}{A}\right)\right| = \frac{|Ax_1 + C|}{|A|} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

(Note that A and B cannot both be zero.) The slope of the line $Ax + By + C = 0$ is $-A/B$.

The equation of the line through (x_1, y_1) perpendicular to $Ax + By + C = 0$ is:

$$\begin{aligned} y - y_1 &= \frac{B}{A}(x - x_1) \\ Ay - Ay_1 &= Bx - Bx_1 \\ Bx_1 - Ay_1 &= Bx - Ay \end{aligned}$$

The point of intersection of these two lines is:

$$\begin{aligned} Ax + By &= -C && \Rightarrow A^2x + ABy = -AC && (1) \\ Bx - Ay &= Bx_1 - Ay_1 && \Rightarrow \underline{B^2x - ABy} = \underline{B^2x_1 - ABy_1} && (2) \\ & && (A^2 + B^2)x = -AC + B^2x_1 - ABy_1 && \text{(By adding equations (1) and (2))} \\ & && x = \frac{-AC + B^2x_1 - ABy_1}{A^2 + B^2} \end{aligned}$$

$$\begin{aligned} Ax + By &= -C && \Rightarrow ABx + B^2y = -BC && (3) \\ Bx - Ay &= Bx_1 - Ay_1 && \Rightarrow \underline{-ABx + A^2y} = \underline{-ABx_1 + A^2y_1} && (4) \\ & && (A^2 + B^2)y = -BC - ABx_1 + A^2y_1 && \text{(By adding equations (3) and (4))} \\ & && y = \frac{-BC - ABx_1 + A^2y_1}{A^2 + B^2} \end{aligned}$$

$\left(\frac{-AC + B^2x_1 - ABy_1}{A^2 + B^2}, \frac{-BC - ABx_1 + A^2y_1}{A^2 + B^2}\right)$ point of intersection

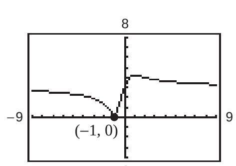
The distance between (x_1, y_1) and this point gives you the distance between (x_1, y_1) and the line $Ax + By + C = 0$.

$$\begin{aligned} d &= \sqrt{\left[\frac{-AC + B^2x_1 - ABy_1}{A^2 + B^2} - x_1\right]^2 + \left[\frac{-BC - ABx_1 + A^2y_1}{A^2 + B^2} - y_1\right]^2} \\ &= \sqrt{\left[\frac{-AC - ABy_1 - A^2x_1}{A^2 + B^2}\right]^2 + \left[\frac{-BC - ABx_1 - B^2y_1}{A^2 + B^2}\right]^2} \\ &= \sqrt{\left[\frac{-A(C + By_1 + Ax_1)}{A^2 + B^2}\right]^2 + \left[\frac{-B(C + Ax_1 + By_1)}{A^2 + B^2}\right]^2} = \sqrt{\frac{(A^2 + B^2)(C + Ax_1 + By_1)^2}{(A^2 + B^2)^2}} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}} \end{aligned}$$

88. $y = mx + 4 \Rightarrow mx + (-1)y + 4 = 0$

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}} = \frac{|m3 + (-1)(1) + 4|}{\sqrt{m^2 + (-1)^2}} = \frac{|3m + 3|}{\sqrt{m^2 + 1}}$$

The distance is 0 when $m = -1$. In this case, the line $y = -x + 4$ contains the point (3, 1).

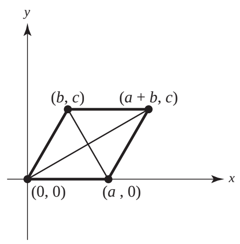


89. For simplicity, let the vertices of the rhombus be (0, 0), $(a, 0)$, (b, c), and $(a + b, c)$, as shown in the figure.

The slopes of the diagonals are then $m_1 = \dfrac{c}{a+b}$ and $m_2 = \dfrac{c}{b-a}$. Because the sides of the rhombus are equal, $a^2 = b^2 + c^2$, and you have

$$m_1 m_2 = \frac{c}{a+b} \cdot \frac{c}{b-a} = \frac{c^2}{b^2 - a^2} = \frac{c^2}{-c^2} = -1.$$

Therefore, the diagonals are perpendicular.



90. For simplicity, let the vertices of the quadrilateral be (0, 0), (a, 0), (b, c), and (d, e), as shown in the figure. The midpoints of the sides are

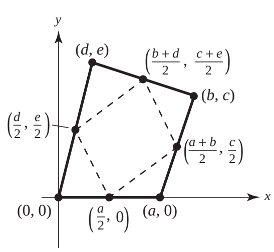
$$\left(\frac{a}{2}, 0\right), \left(\frac{a+b}{2}, \frac{c}{2}\right), \left(\frac{b+d}{2}, \frac{c+e}{2}\right), \text{ and } \left(\frac{d}{2}, \frac{e}{2}\right).$$

The slope of the opposite sides are equal:

$$\frac{\frac{c}{2} - 0}{\frac{a+b}{2} - \frac{a}{2}} = \frac{\frac{c+e}{2} - \frac{e}{2}}{\frac{b+d}{2} - \frac{d}{2}} = \frac{c}{b}$$

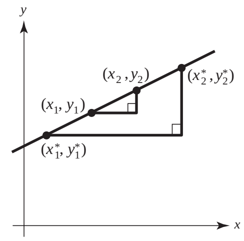
$$\frac{0 - \frac{e}{2}}{\frac{a}{2} - \frac{d}{2}} = \frac{\frac{c}{2} - \frac{c+e}{2}}{\frac{a+b}{2} - \frac{b+d}{2}} = -\frac{e}{a-d}$$

Therefore, the figure is a parallelogram.



91. Consider the figure below in which the four points are collinear. Because the triangles are similar, the result immediately follows.

$$\frac{y_2^* - y_1^*}{x_2^* - x_1^*} = \frac{y_2 - y_1}{x_2 - x_1}$$



92. If $m_1 = -1/m_2$, then $m_1 m_2 = -1$. Let L_3 be a line with slope m_3 that is perpendicular to L_1. Then $m_1 m_3 = -1$. So, $m_2 = m_3 \Rightarrow L_2$ and L_3 are parallel. Therefore, L_2 and L_1 are also perpendicular.

93. True.

$$ax + by = c_1 \Rightarrow y = -\frac{a}{b}x + \frac{c_1}{b} \Rightarrow m_1 = -\frac{a}{b}$$

$$bx - ay = c_2 \Rightarrow y = \frac{b}{a}x - \frac{c_2}{a} \Rightarrow m_2 = \frac{b}{a}$$

$$m_2 = -\frac{1}{m_1}$$

94. False; if m_1 is positive, then $m_2 = -1/m_1$ is negative.

95. True. The slope must be positive.

96. True. The general form $Ax + By + C = 0$ includes both horizontal and vertical lines.

Section 1.3 Functions and Their Graphs

1. (a) $f(0) = 7(0) - 4 = -4$

(b) $f(-3) = 7(-3) - 4 = -25$

(c) $f(b) = 7(b) - 4 = 7b - 4$

(d) $f(x - 1) = 7(x - 1) - 4 = 7x - 11$

2. (a) $f(-4) = \sqrt{-4 + 5} = \sqrt{1} = 1$

(b) $f(11) = \sqrt{11 + 5} = \sqrt{16} = 4$

(c) $f(4) = \sqrt{4 + 5} = \sqrt{9} = 3$

(d) $f(x + \Delta x) = \sqrt{x + \Delta x + 5}$

3. (a) $g(0) = 5 - 0^2 = 5$

(b) $g(\sqrt{5}) = 5 - (\sqrt{5})^2 = 5 - 5 = 0$

(c) $g(-2) = 5 - (-2)^2 = 5 - 4 = 1$

(d) $g(t - 1) = 5 - (t - 1)^2 = 5 - (t^2 - 2t + 1)$

$= 4 + 2t - t^2$

4. (a) $g(4) = 4^2(4 - 4) = 0$

(b) $g\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^2\left(\frac{3}{2} - 4\right) = \frac{9}{4}\left(-\frac{5}{2}\right) = -\frac{45}{8}$

(c) $g(c) = c^2(c - 4) = c^3 - 4c^2$

(d) $g(t + 4) = (t + 4)^2(t + 4 - 4)$

$= (t + 4)^2 t = t^3 + 8t^2 + 16t$

5. (a) $f(0) = \cos(2(0)) = \cos 0 = 1$

(b) $f\left(-\frac{\pi}{4}\right) = \cos\left(2\left(-\frac{\pi}{4}\right)\right) = \cos\left(-\frac{\pi}{2}\right) = 0$

(c) $f\left(\frac{\pi}{3}\right) = \cos\left(2\left(\frac{\pi}{3}\right)\right) = \cos\frac{2\pi}{3} = -\frac{1}{2}$

(d) $f(\pi) = \cos(2(\pi)) = 1$

6. (a) $f(\pi) = \sin \pi = 0$

(b) $f\left(\frac{5\pi}{4}\right) = \sin\left(\frac{5\pi}{4}\right) = \frac{-\sqrt{2}}{2}$

(c) $f\left(\frac{2\pi}{3}\right) = \sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$

(d) $f\left(-\frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$

7. $\frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{(x + \Delta x)^3 - x^3}{\Delta x} = \frac{x^3 + 3x^2\Delta x + 3x^2(\Delta x)^2 + (\Delta x)^3 - x^3}{\Delta x} = 3x^2 + 3x\Delta x + (\Delta x)^2, \ \Delta x \neq 0$

8. $\frac{f(x) - f(1)}{x - 1} = \frac{3x - 1 - (3 - 1)}{x - 1} = \frac{3(x - 1)}{x - 1} = 3, \ x \neq 1$

9. $\frac{f(x) - f(2)}{x - 2} = \frac{\left(1/\sqrt{x - 1} - 1\right)}{x - 2}$

$= \frac{1 - \sqrt{x - 1}}{(x - 2)\sqrt{x - 1}} \cdot \frac{1 + \sqrt{x - 1}}{1 + \sqrt{x - 1}} = \frac{2 - x}{(x - 2)\sqrt{x - 1}\left(1 + \sqrt{x - 1}\right)} = \frac{-1}{\sqrt{x - 1}\left(1 + \sqrt{x - 1}\right)}, \ x \neq 2$

10. $\frac{f(x) - f(1)}{x - 1} = \frac{x^3 - x - 0}{x - 1} = \frac{x(x + 1)(x - 1)}{x - 1} = x(x + 1), \ x \neq 1$

11. $f(x) = 4x^2$

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

12. $g(x) = x^2 - 5$

Domain: $(-\infty, \infty)$

Range: $[-5, \infty)$

13. $f(x) = x^3$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

14. $h(x) = 4 - x^2$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 4]$

15. $g(x) = \sqrt{6x}$

Domain: $6x \geq 0$

$x \geq 0 \Rightarrow [0, \infty)$

Range: $[0, \infty)$

16. $h(x) = -\sqrt{x + 3}$

Domain: $x + 3 \geq 0 \Rightarrow [-3, \infty)$

Range: $(-\infty, 0]$

17. $f(x) = \sqrt{16 - x^2}$

$16 - x^2 \geq 0 \Rightarrow x^2 \leq 16$

Domain: $[-4, 4]$

Range: $[0, 4]$

Note: $y = \sqrt{16 - x^2}$ is a semicircle of radius 4.

18. $f(x) = |x - 3|$

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

19. $f(t) = \sec\frac{\pi t}{4}$

$\frac{\pi t}{4} \neq \frac{(2n+1)\pi}{2} \Rightarrow t \neq 4n + 2$

Domain: all $t \neq 4n + 2$, n an integer

Range: $(-\infty, -1] \cup [1, \infty)$

20. $h(t) = \cot t$

Domain: all $t = n\pi$, n an integer

Range: $(-\infty, \infty)$

21. $f(x) = \frac{3}{x}$

Domain: all $x \neq 0 \Rightarrow (-\infty, 0) \cup (0, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$

22. $f(x) = \frac{x-2}{x+4}$

Domain: all $x \neq -4$

Range: all $y \neq 1$

[**Note:** You can see that the range is all $y \neq 1$ by graphing f.]

23. $f(x) = \sqrt{x} + \sqrt{1-x}$

$x \geq 0$ and $1 - x \geq 0$

$x \geq 0$ and $x \leq 1$

Domain: $0 \leq x \leq 1 \Rightarrow [0, 1]$

24. $f(x) = \sqrt{x^2 - 3x + 2}$

$x^2 - 3x + 2 \geq 0$

$(x-2)(x-1) \geq 0$

Domain: $x \geq 2$ or $x \leq 1$

Domain: $(-\infty, 1] \cup [2, \infty)$

25. $g(x) = \frac{2}{1 - \cos x}$

$1 - \cos x \neq 0$

$\cos x \neq 1$

Domain: all $x \neq 2n\pi$, n an integer

26. $h(x) = \frac{1}{\sin x - (1/2)}$

$\sin x - \frac{1}{2} \neq 0$

$\sin x \neq \frac{1}{2}$

Domain: all $x \neq \frac{\pi}{6} + 2n\pi, \frac{5\pi}{6} + 2n\pi$, n integer

27. $f(x) = \frac{1}{|x+3|}$

$|x + 3| \neq 0$

$x + 3 \neq 0$

Domain: all $x \neq -3$

Domain: $(-\infty, -3) \cup (-3, \infty)$

28. $g(x) = \frac{1}{|x^2 - 4|}$

$|x^2 - 4| \neq 0$

$(x-2)(x+2) \neq 0$

Domain: all $x \neq \pm 2$

Domain: $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

29. $f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \geq 0 \end{cases}$

(a) $f(-1) = 2(-1) + 1 = -1$

(b) $f(0) = 2(0) + 2 = 2$

(c) $f(2) = 2(2) + 2 = 6$

(d) $f(t^2 + 1) = 2(t^2 + 1) + 2 = 2t^2 + 4$

(**Note:** $t^2 + 1 \geq 0$ for all t.)

Domain: $(-\infty, \infty)$

Range: $(-\infty, 1) \cup [2, \infty)$

30. $f(x) = \begin{cases} x^2 + 2, & x \leq 1 \\ 2x^2 + 2, & x > 1 \end{cases}$

(a) $f(-2) = (-2)^2 + 2 = 6$

(b) $f(0) = 0^2 + 2 = 2$

(c) $f(1) = 1^2 + 2 = 3$

(d) $f(s^2 + 2) = 2(s^2 + 2)^2 + 2 = 2s^4 + 8s^2 + 10$

(**Note:** $s^2 + 2 > 1$ for all s.)

Domain: $(-\infty, \infty)$

Range: $[2, \infty)$

31. $f(x) = \begin{cases} |x| + 1, & x < 1 \\ -x + 1, & x \geq 1 \end{cases}$

(a) $f(-3) = |-3| + 1 = 4$

(b) $f(1) = -1 + 1 = 0$

(c) $f(3) = -3 + 1 = -2$

(d) $f(b^2 + 1) = -(b^2 + 1) + 1 = -b^2$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 0] \cup [1, \infty)$

32. $f(x) = \begin{cases} \sqrt{x + 4}, & x \leq 5 \\ (x - 5)^2, & x > 5 \end{cases}$

(a) $f(-3) = \sqrt{-3 + 4} = \sqrt{1} = 1$

(b) $f(0) = \sqrt{0 + 4} = 2$

(c) $f(5) = \sqrt{5 + 4} = 3$

(d) $f(10) = (10 - 5)^2 = 25$

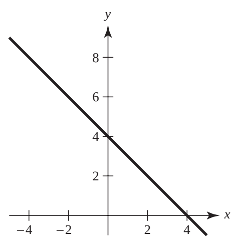
Domain: $[-4, \infty)$

Range: $[0, \infty)$

33. $f(x) = 4 - x$

Domain: $(-\infty, \infty)$

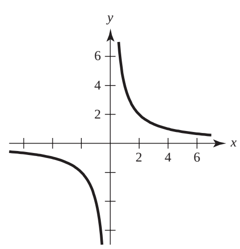
Range: $(-\infty, \infty)$



34. $g(x) = \dfrac{4}{x}$

Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$



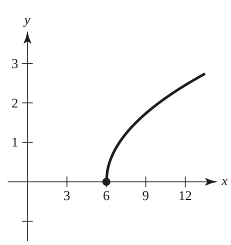
35. $h(x) = \sqrt{x - 6}$

Domain:

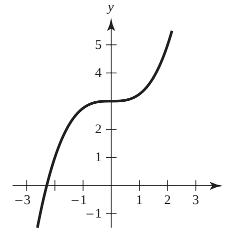
$x - 6 \geq 0$

$x \geq 6 \Rightarrow [6, \infty)$

Range: $[0, \infty)$



36. $f(x) = \frac{1}{4}x^3 + 3$



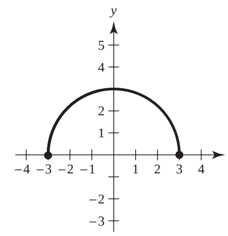
Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

37. $f(x) = \sqrt{9 - x^2}$

Domain: $[-3, 3]$

Range: $[0, 3]$



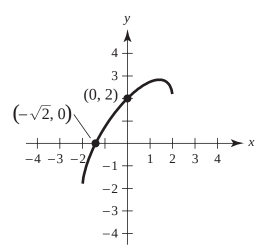
38. $f(x) = x + \sqrt{4 - x^2}$

Domain: $[-2, 2]$

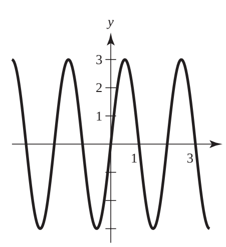
Range: $\left[-2, 2\sqrt{2}\right] \approx [-2, 2.83]$

y-intercept: $(0, 2)$

x-intercept: $\left(-\sqrt{2}, 0\right)$



39. $g(t) = 3 \sin \pi t$



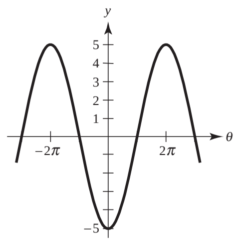
Domain: $(-\infty, \infty)$

Range: $[-3, 3]$

40. $h(\theta) = -5\cos\dfrac{\theta}{2}$

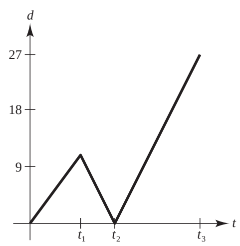
Domain: $(-\infty, \infty)$

Range: $[-5, 5]$



41. The student travels $\dfrac{2-0}{4-0} = \dfrac{1}{2}$ mi/min during the first 4 minutes. The student is stationary for the next 2 minutes. Finally, the student travels $\dfrac{6-2}{10-6} = 1$ mi/min during the final 4 minutes.

42.



43. $x - y^2 = 0 \Rightarrow y = \pm\sqrt{x}$

y is not a function of x. Some vertical lines intersect the graph twice.

44. $\sqrt{x^2-4} - y = 0 \Rightarrow y = \sqrt{x^2-4}$

y is a function of x. Vertical lines intersect the graph at most once.

45. y is a function of x. Vertical lines intersect the graph at most once.

46. $x^2 + y^2 = 4$

$$y = \pm\sqrt{4 - x^2}$$

y is not a function of x. Some vertical lines intersect the graph twice.

47. $x^2 + y^2 = 16 \Rightarrow y = \pm\sqrt{16 - x^2}$

y is not a function of x because there are two values of y for some x.

48. $x^2 + y = 16 \Rightarrow y = 16 - x^2$

y is a function of x because there is one value of y for each x.

49. $y^2 = x^2 - 1 \Rightarrow y = \pm\sqrt{x^2 - 1}$

y is not a function of x because there are two values of y for some x.

50. $x^2y - x^2 + 4y = 0 \Rightarrow y = \dfrac{x^2}{x^2 + 4}$

y is a function of x because there is one value of y for each x.

51. The transformation is a horizontal shift two units to the right.

Shifted function: $y = \sqrt{x - 2}$

52. The transformation is a vertical shift 4 units upward.

Shifted function: $y = \sin x + 4$

53. The transformation is a horizontal shift 2 units to the right and a vertical shift 1 unit downward.

Shifted function: $y = (x-2)^2 - 1$

54. The transformation is a horizontal shift 1 unit to the left and a vertical shift 2 units upward.

Shifted function: $y = (x+1)^3 + 2$

55. $y = f(x+5)$ is a horizontal shift 5 units to the left. Matches d.

56. $y = f(x) - 5$ is a vertical shift 5 units downward. Matches b.

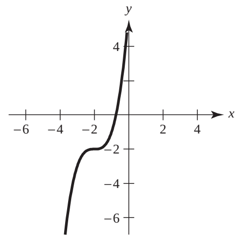
57. $y = -f(-x) - 2$ is a reflection in the y-axis, a reflection in the x-axis, and a vertical shift downward 2 units. Matches c.

58. $y = -f(x-4)$ is a horizontal shift 4 units to the right, followed by a reflection in the x-axis. Matches a.

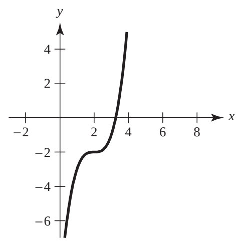
59. $y = f(x+6) + 2$ is a horizontal shift to the left 6 units, and a vertical shift upward 2 units. Matches e.

60. $y = f(x-1) + 3$ is a horizontal shift to the right 1 unit, and a vertical shift upward 3 units. Matches g.

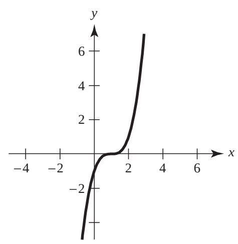
61. (a) The graph is shifted 3 units to the left.



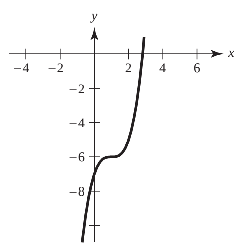
(b) The graph is shifted 1 unit to the right.



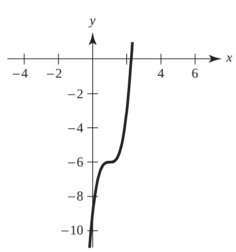
(c) The graph is shifted 2 units upward.



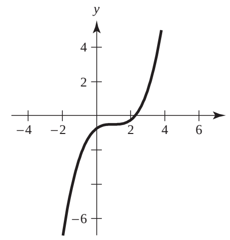
(d) The graph is shifted 4 units downward.



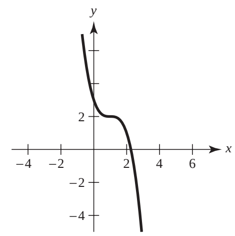
(e) The graph is stretched vertically by a factor of 3.



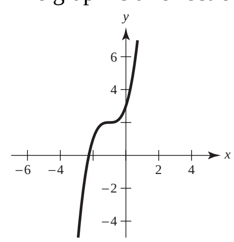
(f) The graph is stretched vertically by a factor of $\frac{1}{4}$.



(g) The graph is a reflection in the x-axis.



(h) The graph is a reflection about the origin.

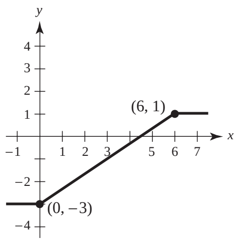


62. (a) $g(x) = f(x - 4)$

$g(6) = f(2) = 1$

$g(0) = f(-4) = -3$

The graph is shifted 4 units to the right.

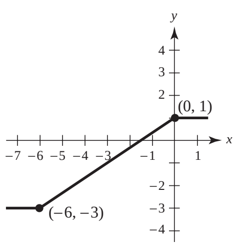


(b) $g(x) = f(x + 2)$

$g(0) = f(2) = 1$

$g(-6) = f(-4) = -3$

The graph is shifted 2 units to the left.

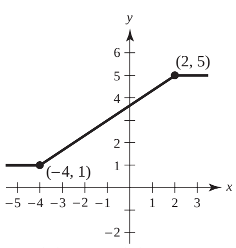


(c) $g(x) = f(x) + 4$

$g(2) = f(2) + 4 = 5$

$g(-4) = f(-4) + 4 = 1$

The graph is shifted 4 units upward.

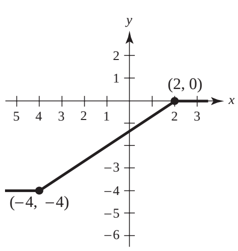


(d) $g(x) = f(x) - 1$

$g(2) = f(2) - 1 = 0$

$g(-4) = f(-4) - 1 = -4$

The graph is shifted 1 unit downward.

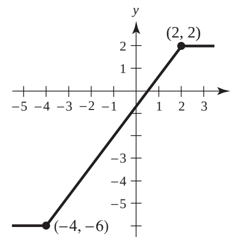


(e) $g(x) = 2f(x)$

$g(2) = 2f(2) = 2$

$g(-4) = 2f(-4) = -6$

The graph is stretched vertically by a factor of 2.

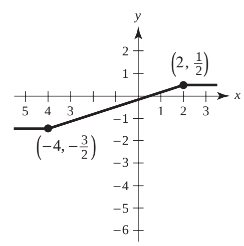


(f) $g(x) = \frac{1}{2}f(x)$

$g(2) = \frac{1}{2}f(2) = \frac{1}{2}$

$g(-4) = \frac{1}{2}f(-4) = -\frac{3}{2}$

The graph is stretched vertically by a factor of $\frac{1}{2}$.

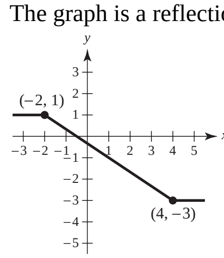


(g) $g(x) = f(-x)$

$g(-2) = f(2) = 1$

$g(4) = f(-4) = -3$

The graph is a reflection in the *y*-axis.

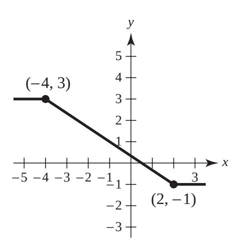


(h) $g(x) = -f(x)$

$g(2) = f(2) = -1$

$g(-4) = f(-4) = 3$

The graph is a reflection in the *x*-axis.



63. $f(x) = 3x - 4, \quad g(x) = 4$

(a) $f(x) + g(x) = (3x - 4) + 4 = 3x$

(b) $f(x) - g(x) = (3x - 4) - 4 = 3x - 8$

(c) $f(x) \cdot g(x) = (3x - 4)(4) = 12x - 16$

(d) $f(x)/g(x) = \dfrac{3x - 4}{4} = \dfrac{3}{4}x - 1$

64. $f(x) = x^2 + 5x + 4, \quad g(x) = x + 1$

(a) $f(x) + g(x) = (x^2 + 5x + 4) + (x + 1) = x^2 + 6x + 5$

(b) $f(x) - g(x) = (x^2 + 5x + 4) - (x + 1) = x^2 + 4x + 3$

(c) $f(x) \cdot g(x) = (x^2 + 5x + 4)(x + 1)$

$= x^3 + 5x^2 + 4x + x^2 + 5x + 4$

$= x^3 + 6x^2 + 9x + 4$

(d) $f(x)/g(x) = \dfrac{x^2 + 5x + 4}{x + 1} = \dfrac{(x + 4)(x + 1)}{x + 1} = x + 4, x \neq -1$

65. (a) $f(g(1)) = f(0) = 0$

(b) $g(f(1)) = g(1) = 0$

(c) $g(f(0)) = g(0) = -1$

(d) $f(g(-4)) = f(15) = \sqrt{15}$

(e) $f(g(x)) = f(x^2 - 1) = \sqrt{x^2 - 1}$

(f) $g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^2 - 1 = x - 1, \ (x \geq 0)$

66. $f(x) = \sin x, \ g(x) = \pi x$

(a) $f(g(2)) = f(2\pi) = \sin(2\pi) = 0$

(b) $f\left(g\left(\dfrac{1}{2}\right)\right) = f\left(\dfrac{\pi}{2}\right) = \sin\left(\dfrac{\pi}{2}\right) = 1$

(c) $g(f(0)) = g(0) = 0$

(d) $g\left(f\left(\dfrac{\pi}{4}\right)\right) = g\left(\sin\left(\dfrac{\pi}{4}\right)\right)$

$= g\left(\dfrac{\sqrt{2}}{2}\right) = \pi\left(\dfrac{\sqrt{2}}{2}\right) = \dfrac{\pi\sqrt{2}}{2}$

(e) $f(g(x)) = f(\pi x) = \sin(\pi x)$

(f) $g(f(x)) = g(\sin x) = \pi \sin x$

67. $f(x) = x^2, \ g(x) = \sqrt{x}$

$(f \circ g)(x) = f(g(x))$

$= f(\sqrt{x}) = (\sqrt{x})^2 = x, \ x \geq 0$

Domain: $[0, \infty)$

$(g \circ f)(x) = g(f(x)) = g(x^2) = \sqrt{x^2} = |x|$

Domain: $(-\infty, \infty)$

No. Their domains are different. $(f \circ g) = (g \circ f)$ for $x \geq 0$.

68. $f(x) = x^2 - 1, \ g(x) = \cos x$

$(f \circ g)(x) = f(g(x)) = f(\cos x) = \cos^2 x - 1$

Domain: $(-\infty, \infty)$

$(g \circ f)(x) = g(x^2 - 1) = \cos(x^2 - 1)$

Domain: $(-\infty, \infty)$

No, $f \circ g \neq g \circ f$.

69. $f(x) = \frac{3}{x},\ g(x) = x^2 - 1$

$(f \circ g)(x) = f(g(x)) = f(x^2 - 1) = \frac{3}{x^2 - 1}$

Domain: all $x \neq \pm 1 \Rightarrow (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

$(g \circ f)(x) = g(f(x))$

$= g\left(\frac{3}{x}\right) = \left(\frac{3}{x}\right)^2 - 1 = \frac{9}{x^2} - 1 = \frac{9 - x^2}{x^2}$

Domain: all $x \neq 0 \Rightarrow (-\infty, 0) \cup (0, \infty)$

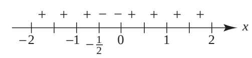
No, $f \circ g \neq g \circ f$.

70. $(f \circ g)(x) = f\left(\sqrt{x + 2}\right) = \frac{1}{\sqrt{x + 2}}$

Domain: $(-2, \infty)$

$(g \circ f)(x) = g\left(\frac{1}{x}\right) = \sqrt{\frac{1}{x} + 2} = \sqrt{\frac{1 + 2x}{x}}$

You can find the domain of $g \circ f$ by determining the intervals where $(1 + 2x)$ and x are both positive, or both negative.



Domain: $\left(-\infty, -\frac{1}{2}\right], (0, \infty)$

71. (a) $(f \circ g)(3) = f(g(3)) = f(-1) = 4$

(b) $g(f(2)) = g(1) = -2$

(c) $g(f(5)) = g(-5)$, which is undefined

(d) $(f \circ g)(-3) = f(g(-3)) = f(-2) = 3$

(e) $(g \circ f)(-1) = g(f(-1)) = g(4) = 2$

(f) $f(g(-1)) = f(-4)$, which is undefined

72. $(A \circ r)(t) = A(r(t)) = A(0.6t) = \pi(0.6t)^2 = 0.36\pi t^2$

$(A \circ r)(t)$ represents the area of the circle at time t.

73. $F(x) = \sqrt{2x - 2}$

Let $h(x) = 2x,\ g(x) = x - 2$ and $f(x) = \sqrt{x}$.

Then, $(f \circ g \circ h)(x) = f(g(2x)) = f((2x) - 2) = \sqrt{(2x) - 2} = \sqrt{2x - 2} = F(x)$.

[Other answers possible]

74. $F(x) = -4 \sin(1 - x)$

Let $f(x) = -4x,\ g(x) = \sin x$ and $h(x) = 1 - x$. Then,

$(f \circ g \circ h)(x) = f(g(1 - x)) = f(\sin(1 - x)) = -4 \sin(1 - x) = F(x)$.

[Other answers possible]

75. (a) If f is even, then $\left(\frac{3}{2}, 4\right)$ is on the graph.

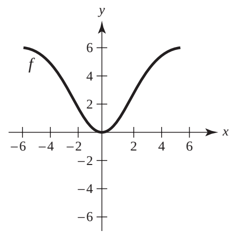
(b) If f is odd, then $\left(\frac{3}{2}, -4\right)$ is on the graph.

76. (a) If f is even, then $(-4, 9)$ is on the graph.

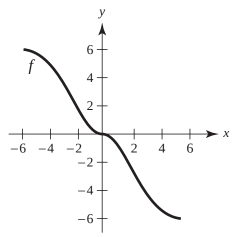
(b) If f is odd, then $(-4, -9)$ is on the graph.

77. f is even because the graph is symmetric about the y-axis. g is neither even nor odd. h is odd because the graph is symmetric about the origin.

78. (a) If f is even, then the graph is symmetric about the y-axis.



(b) If f is odd, then the graph is symmetric about the origin.



79. $f(x) = x^2(4 - x^2)$

$f(-x) = (-x)^2(4 - (-x)^2) = x^2(4 - x^2) = f(x)$

f is even.

$f(x) = x^2(4 - x^2) = 0$

$x^2(2 - x)(2 + x) = 0$

Zeros: $x = 0, -2, 2$

80. $f(x) = \sqrt[3]{x}$

$f(-x) = \sqrt[3]{(-x)} = -\sqrt[3]{x} = -f(x)$

f is odd.

$f(x) = \sqrt[3]{x} = 0 \Rightarrow x = 0$ is the zero.

81. $f(x) = x \cos x$

$f(-x) = (-x)\cos(-x) = -x \cos x = -f(x)$

f is odd.

$f(x) = x \cos x = 0$

Zeros: $x = 0, \dfrac{\pi}{2} + n\pi$, where n is an integer

82. $f(x) = \sin^2 x$

$f(-x) = \sin^2(-x) = \sin(-x)\sin(-x) = (-\sin x)(-\sin x) = \sin^2 x$

f is even.

$\sin^2 x = 0 \Rightarrow \sin x = 0$

Zeros: $x = n\pi$, where n is an integer

83. Slope $= \dfrac{4 - (-6)}{-2 - 0} = \dfrac{10}{-2} = -5$

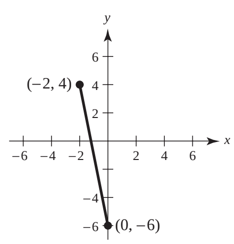
$y - 4 = -5(x - (-2))$

$y - 4 = -5x - 10$

$y = -5x - 6$

For the line segment, you must restrict the domain.

$f(x) = -5x - 6, \; -2 \le x \le 0$



84. Slope $= \dfrac{8 - 1}{5 - 3} = \dfrac{7}{2}$

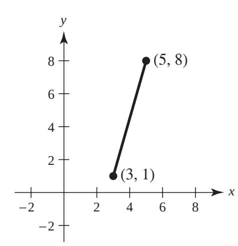
$y - 1 = \dfrac{7}{2}(x - 3)$

$y - 1 = \dfrac{7}{2}x - \dfrac{21}{2}$

$y = \dfrac{7}{2}x - \dfrac{19}{2}$

For the line segment, you must restrict the domain.

$f(x) = \dfrac{7}{2}x - \dfrac{19}{2}, \; 3 \le x \le 5$

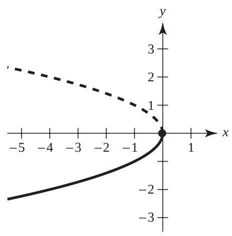


85. $x + y^2 = 0$

$y^2 = -x$

$y = -\sqrt{-x}$

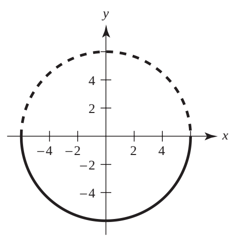
$f(x) = -\sqrt{-x},\ x \le 0$



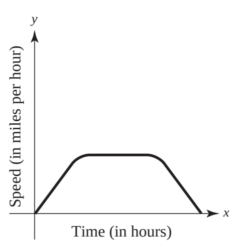
86. $x^2 + y^2 = 36$

$y^2 = 36 - x^2$

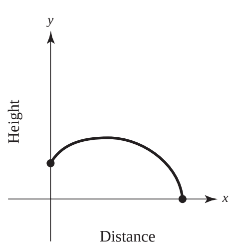
$y = -\sqrt{36 - x^2},\ -6 \le x \le 6$



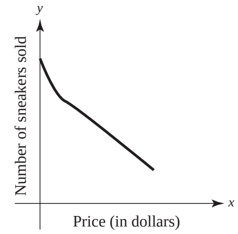
87. Answers will vary. *Sample answer*: Speed begins and ends at 0. The speed might be constant in the middle:



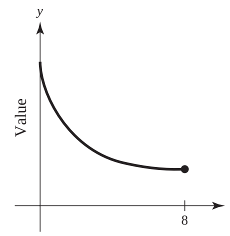
88. Answers will vary. *Sample answer*: Height begins a few feet above 0, and ends at 0.



89. Answers will vary. *Sample answer*: In general, as the price decreases, the store will sell more.



90. Answers will vary. *Sample answer*: As time goes on, the value of the car will decrease



91. $y = \sqrt{c - x^2}$

$y^2 = c - x^2$

$x^2 + y^2 = c$, a circle.

For the domain to be $[-5, 5]$, $c = 25$.

92. For the domain to be the set of all real numbers, you must require that $x^2 + 3cx + 6 \ne 0$. So, the discriminant must be less than zero:

$(3c)^2 - 4(6) < 0$

$9c^2 < 24$

$c^2 < \frac{8}{3}$

$-\sqrt{\frac{8}{3}} < c < \sqrt{\frac{8}{3}}$

$-\frac{2}{3}\sqrt{6} < c < \frac{2}{3}\sqrt{6}$

93. (a) $T(4) = 16°$, $T(15) \approx 23°$

(b) If $H(t) = T(t - 1)$, then the changes in temperature will occur 1 hour later.

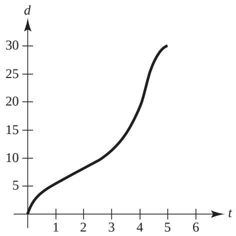
(c) If $H(t) = T(t) - 1$, then the overall temperature would be 1 degree lower.

94. (a) For each time t, there corresponds a depth d.

(b) Domain: $0 \le t \le 5$

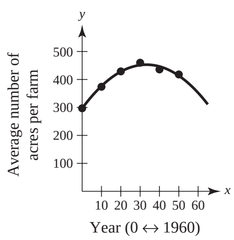
Range: $0 \le d \le 30$

(c)



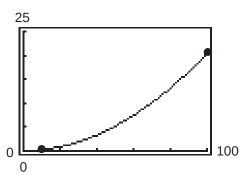
(d) $d(4) \approx 18$. At time 4 seconds, the depth is approximately 18 cm.

95. (a)



(b) $A(25) \approx 445$ (Answers will vary.)

96. (a)



(b) $H\left(\frac{x}{1.6}\right) = 0.002\left(\frac{x}{1.6}\right)^2 + 0.005\left(\frac{x}{1.6}\right) - 0.029$

$= 0.00078125x^2 + 0.003125x - 0.029$

97. $f(x) = |x| + |x - 2|$

If $x < 0$, then $f(x) = -x - (x - 2) = -2x + 2$.

If $0 \le x < 2$, then $f(x) = x - (x - 2) = 2$.

If $x \ge 2$, then $f(x) = x + (x - 2) = 2x - 2$.

So,

$$f(x) = \begin{cases} -2x + 2, & x \le 0 \\ 2, & 0 < x < 2 \\ 2x - 2, & x \ge 2 \end{cases}.$$

98. $p_1(x) = x^3 - x + 1$ has one zero. $p_2(x) = x^3 - x$ has three zeros. Every cubic polynomial has at least one zero. Given $p(x) = Ax^3 + Bx^2 + Cx + D$, you have $p \to -\infty$ as $x \to -\infty$ and $p \to \infty$ as $x \to \infty$ if $A > 0$. Furthermore, $p \to \infty$ as $x \to -\infty$ and $p \to -\infty$ as $x \to \infty$ if $A < 0$. Because the graph has no breaks, the graph must cross the x-axis at least one time.

99. $f(-x) = a_{2n+1}(-x)^{2n+1} + \cdots + a_3(-x)^3 + a_1(-x)$

$= -\left[a_{2n+1}x^{2n+1} + \cdots + a_3x^3 + a_1x\right]$

$= -f(x)$

Odd

100. $f(-x) = a_{2n}(-x)^{2n} + a_{2n-2}(-x)^{2n-2} + \cdots + a_2(-x)^2 + a_0$

$= a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0$

$= f(x)$

Even

101. Let $F(x) = f(x)g(x)$ where f and g are even. Then $F(-x) = f(-x)g(-x) = f(x)g(x) = F(x)$.

So, $F(x)$ is even. Let $F(x) = f(x)g(x)$ where f and g are odd. Then

$F(-x) = f(-x)g(-x) = [-f(x)][-g(x)] = f(x)g(x) = F(x)$.

So, $F(x)$ is even.

102. Let $F(x) = f(x)g(x)$ where f is even and g is odd. Then

$F(-x) = f(-x)g(-x) = f(x)[-g(x)] = -f(x)g(x) = -F(x)$.

So, $F(x)$ is odd.

103. By equating slopes, $\frac{y-2}{0-3} = \frac{0-2}{x-3}$

$$y - 2 = \frac{6}{x-3}$$

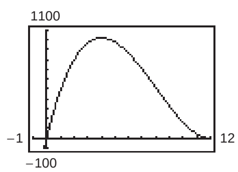
$$y = \frac{6}{x-3} + 2 = \frac{2x}{x-3},$$

$$L = \sqrt{x^2 + y^2} = \sqrt{x^2 + \left(\frac{2x}{x-3}\right)^2}.$$

104. (a) $V = x(24 - 2x)^2$

Domain: $0 < x < 12$

(b)



Maximum volume occurs at $x = 4$. So, the dimensions of the box would be $4 \times 16 \times 16$ cm.

(c)

x	*length and width*	*volume*
1	$24 - 2(1)$	$1[24 - 2(1)]^2 = 484$
2	$24 - 2(2)$	$2[24 - 2(2)]^2 = 800$
3	$24 - 2(3)$	$3[24 - 2(3)]^2 = 972$
4	$24 - 2(4)$	$4[24 - 2(4)]^2 = 1024$
5	$24 - 2(5)$	$5[24 - 2(5)]^2 = 980$
6	$24 - 2(6)$	$6[24 - 2(6)]^2 = 864$

The dimensions of the box that yield a maximum volume appear to be $4 \times 16 \times 16$ cm.

105. False. If $f(x) = x^2$, then $f(-3) = f(3) = 9$, but $-3 \neq 3$.

106. True

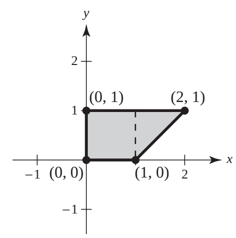
107. True. The function is even.

108. False. If $f(x) = x^2$ then, $f(3x) = (3x)^2 = 9x^2$ and $3f(x) = 3x^2$. So, $3f(x) \neq f(3x)$.

109. False. The constant function $f(x) = 0$ has symmetry with respect to the x-axis.

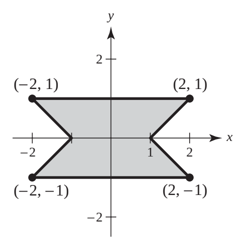
110. True. If the domain is $\{a\}$, then the range is $\{f(a)\}$.

111. First consider the portion of R in the first quadrant: $x \geq 0$, $0 \leq y \leq 1$ and $x - y \leq 1$; shown below.



The area of this region is $1 + \frac{1}{2} = \frac{3}{2}$.

By symmetry, you obtain the entire region R:



The area of R is $4\left(\frac{3}{2}\right) = 6$.

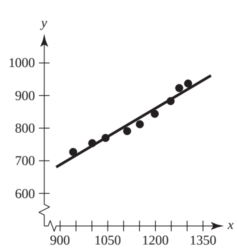
112. Let $g(x) = c$ be constant polynomial.

Then $f(g(x)) = f(c)$ and $g(f(x)) = c$.

So, $f(c) = c$. Because this is true for all real numbers c, f is the identity function: $f(x) = x$.

Section 1.4 Fitting Models to Data

1. (a) and (b)

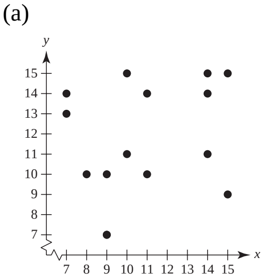


Yes, the data appear to be approximately linear.

The data can be modeled by equation $y = 0.6x + 150$. (Answers will vary).

(c) When $x = 1075$, $y = 0.6(1075) + 150 = 795$.

2. (a)

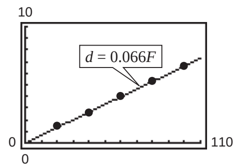


The data do not appear to be linear.

(b) Quiz scores are dependent on several variables such as study time, class attendance, and so on. These variables may change from one quiz to the next.

3. (a) $d = 0.066F$

(b)

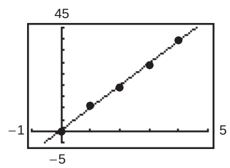


The model fits the data well.

(c) If $F = 55$, then $d \approx 0.066(55) = 3.63$ cm.

4. (a) $s = 9.7t + 0.4$

(b)



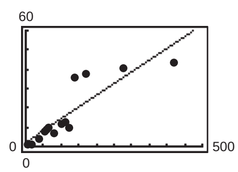
The model fits the data well.

(c) If $t = 2.5$, $s = 24.65$ meters/second.

5. (a) Using a graphing utility, $y = 0.122x + 2.07$

The correlation coefficient is $r \approx 0.87$.

(b)



(c) Greater per capita energy consumption by a country tends to correspond to greater per capita gross national income. The three countries that most differ from the linear model are Canada, Japan, and Italy.

(d) Using a graphing utility, the new model is $y = 0.142x - 1.66$.

The correlation coefficient is $r \approx 0.97$.

6. (a) Trigonometric function

(b) Quadratic function

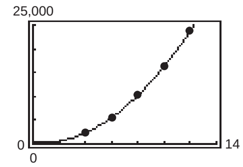
(c) No relationship

(d) Linear function

7. (a) Using graphing utility,

$S = 180.89x^2 - 205.79x + 272.$

(b)



(c) When $x = 2$, $S \approx 583.98$ pounds.

(d) $\dfrac{2370}{584} \approx 4.06$

The breaking strength is approximately 4 times greater.

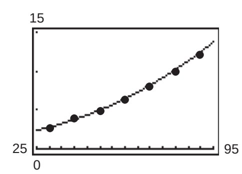
(e) $\dfrac{23,860}{5460} \approx 4.37$

When the height is doubled, the breaking strength increases approximately by a factor of 4.

8. (a) Using a graphing utility

$t = 0.0013s^2 + 0.005s + 1.48.$

(b)



(c) According to the model, the times required to attain speeds of less than 20 miles per hour are all about the same. Furthermore, it takes 1.48 seconds to reach 0 miles per hour, which does not make sense.

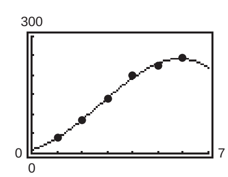
(d) Adding $(0, 0)$ to the data produces

$t = 0.0009s^2 + 0.053s + 0.10.$

(e) Yes. Now the car starts at rest.

9. (a) $y = -1.806x^3 + 14.58x^2 + 16.4x + 10$

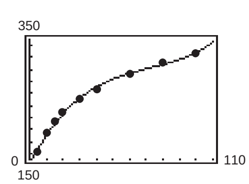
(b)



(c) If $x = 4.5$, $y \approx 214$ horsepower.

10. (a) $T = 2.9856 \times 10^{-4} p^3 - 0.0641p^2 + 5.282p + 143.1$

(b)



(c) For $T = 300°F$, $p \approx 68.29$ lb/in.^{2}.

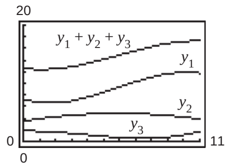
(d) The model is based on data up to 100 pounds per square inch.

11. (a) $y_1 = -0.0172t^3 + 0.305t^2 - 0.87t + 7.3$

$y_2 = -0.038t^2 + 0.45t + 3.5$

$y_3 = 0.0063t^3 + -0.072t^2 + 0.02t + 1.8$

(b)



$y_1 + y_2 + y_3 = -0.0109t^3 + 0.195t^2 - 0.40t + 12.6$

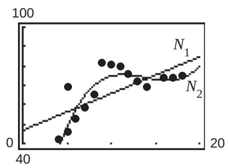
For 2014, $t = 14$. So,

$$y_1 + y_2 + y_3 = -0.0109(14)^3 + 0.195(14)^2 - 0.40(14) + 12.6$$
$$\approx 15.31 \text{ cents/mile}$$

12. (a) $N_1 = 1.89t + 46.8$ Linear model

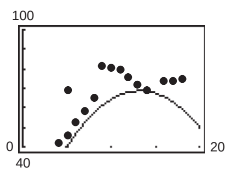
$N_2 = 0.0485t^3 - 2.015t^2 + 27.00t - 42.3$ Cubic model

(b)



(c) The cubic model is the better model.

(d) $N_3 = -0.414t^2 + 11.00t + 4.4$ Quadratic model



The model does not fit the data well.

(e) For 2014, $t = 24$ and

$N_1 \approx 92.16$ million

$N_2 \approx 115.524$ million

The linear model seems too high. The cubic model is better.

(f) Answers will vary.

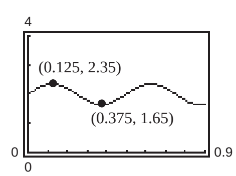
13. (a) Yes, y is a function of t. At each time t, there is one and only one displacement y.

(b) The amplitude is approximately $(2.35 - 1.65)/2 = 0.35$.

The period is approximately $2(0.375 - 0.125) = 0.5$.

(c) One model is $y = 0.35 \sin(4\pi t) + 2$.

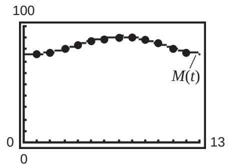
(d)



The model appears to fit the data.

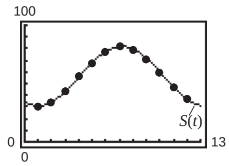
14. (a) $S(t) = 56.37 + 25.47\sin(0.5080t - 2.07)$

(b)



The model is a good fit.

(c)



The model is a good fit.

(d) The average is the constant term in each model. 83.70°F for Miami and 56.37°F for Syracuse.

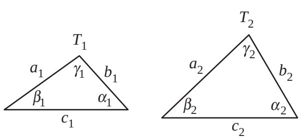
(e) The period for Miami is $2\pi/0.4912 \approx 12.8$. The period for Syracuse is $2\pi/0.5080 \approx 12.4$. In both cases the period is approximately 12, or one year.

(f) Syracuse has greater variability because $25.47 > 7.46$.

15. Answers will vary.

16. Answers will vary.

17. Yes, $A_1 \le A_2$. To see this, consider the two triangles of areas A_1 and A_2:



For $i = 1, 2$, the angles satisfy $\alpha i + \beta i + \gamma i = \pi$. At least one of $\alpha_1 \le \alpha_2,\ \beta_1 \le \beta_2,\ \gamma_1 \le \gamma_2$ must hold. Assume $\alpha_1 \le \alpha_2$. Because $\alpha_2 \le \pi/2$ (acute triangle), and the sine function increases on $[0, \pi/2]$, you have

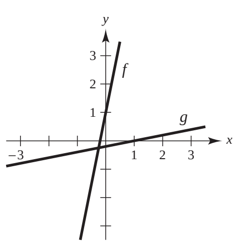
$$A_1 = \tfrac{1}{2}b_1c_1 \sin\alpha_1 \le \tfrac{1}{2}b_2c_2\sin\alpha_1$$
$$\le \tfrac{1}{2}b_2c_2\sin\alpha_2 = A_2$$

Section 1.5 Inverse Functions

1. (a)

$$f(x) = 5x + 1$$
$$g(x) = \frac{x-1}{5}$$
$$f(g(x)) = f\left(\frac{x-1}{5}\right) = 5\left(\frac{x-1}{5}\right) + 1 = x$$
$$g(f(x)) = g(5x+1) = \frac{(5x+1)-1}{5} = x$$

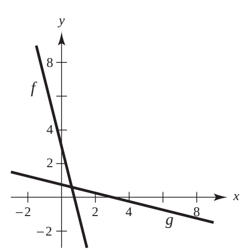
(b)



2. (a)

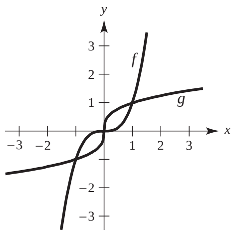
$$f(x) = 3 - 4x$$
$$g(x) = \frac{3-x}{4}$$
$$f(g(x)) = f\left(\frac{3-x}{4}\right) = 3 - 4\left(\frac{3-x}{4}\right) = x$$
$$g(f(x)) = g(3-4x) = \frac{3-(3-4x)}{4} = x$$

(b)



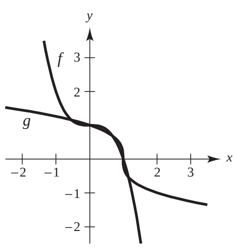
3. (a)
$$f(x) = x^3$$
$$g(x) = \sqrt[3]{x}$$
$$f(g(x)) = f\left(\sqrt[3]{x}\right) = \left(\sqrt[3]{x}\right)^3 = x$$
$$g(f(x)) = g\left(x^3\right) = \sqrt[3]{x^3} = x$$

(b)



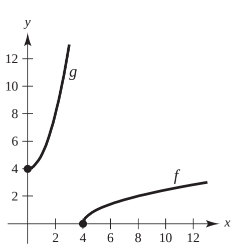
4. (a)
$$f(x) = 1 - x^3$$
$$g(x) = \sqrt[3]{1-x}$$
$$f(g(x)) = f\left(\sqrt[3]{1-x}\right) = 1 - \left(\sqrt[3]{1-x}\right)^3$$
$$= 1 - (1 - x) = x$$
$$g(f(x)) = g\left(1 - x^3\right)$$
$$= \sqrt[3]{1 - \left(1 - x^3\right)} = \sqrt[3]{x^3} = x$$

(b)



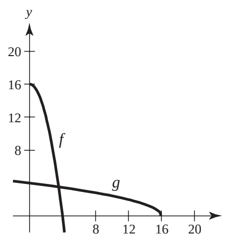
5. (a)
$$f(x) = \sqrt{x-4}$$
$$g(x) = x^2 + 4, \quad x \ge 0$$
$$f(g(x)) = f\left(x^2 + 4\right)$$
$$= \sqrt{\left(x^2 + 4\right) - 4} = \sqrt{x^2} = x$$
$$g(f(x)) = g\left(\sqrt{x-4}\right)$$
$$= \left(\sqrt{x-4}\right)^2 + 4 = x - 4 + 4 = x$$

(b)



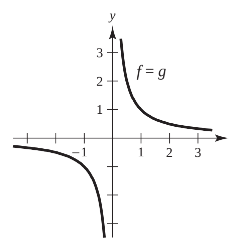
6. (a)
$$f(x) = 16 - x^2, \quad x \ge 0$$
$$g(x) = \sqrt{16 - x}$$
$$f(g(x)) = f\left(\sqrt{16-x}\right) = 16 - \left(\sqrt{16-x}\right)^2$$
$$= 16 - (16 - x) = x$$
$$g(f(x)) = g\left(16 - x^2\right) = \sqrt{16 - \left(16 - x^2\right)}$$
$$= \sqrt{x^2} = x$$

(b)



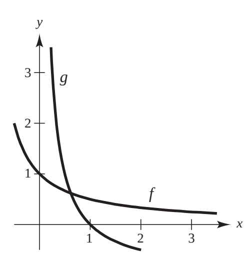
7. (a)
$$f(x) = \frac{1}{x}$$
$$g(x) = \frac{1}{x}$$
$$f(g(x)) = \frac{1}{1/x} = x$$
$$g(f(x)) = \frac{1}{1/x} = x$$

(b)



8. (a)
$$f(x) = \frac{1}{1+x}, \qquad x \ge 0$$
$$g(x) = \frac{1-x}{x}, \qquad 0 < x \le 1$$
$$f(g(x)) = f\left(\frac{1-x}{x}\right) = \frac{1}{1 + \dfrac{1-x}{x}} = \frac{1}{\dfrac{1}{x}} = x$$
$$g(f(x)) = g\left(\frac{1}{1+x}\right) = \frac{1 - \dfrac{1}{1+x}}{\dfrac{1}{1+x}} = \frac{x}{1+x} \cdot \frac{1+x}{1} = x$$

(b)



9. Matches (c)

10. Matches (b)

11. Matches (a)

12. Matches (d)

13. $f(x) = \frac{3}{4}x + 6$

One-to-one; has an inverse

14. $f(x) = 5x - 3$

One-to-one; has an inverse

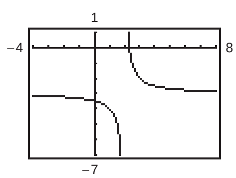
15. $f(\theta) = \sin\theta$

Not one-to-one; does not have an inverse

16. $f(x) = \dfrac{x^2}{x^2 + 4}$

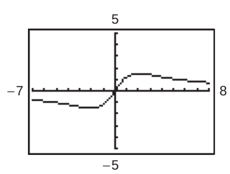
Not one-to-one; does not have an inverse

17. $h(s) = \dfrac{1}{s - 2} - 3$



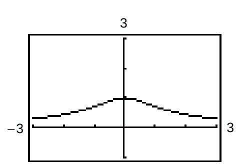
One-to-one; has an inverse

18. $f(x) = \dfrac{6x}{x^2 + 4}$



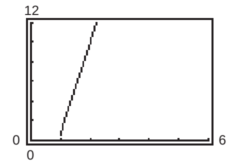
Not one-to-one; does not have an inverse

19. $g(t) = \dfrac{1}{\sqrt{t^2 + 1}}$



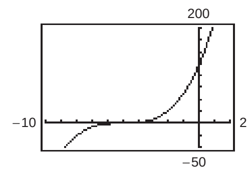
Not one-to-one; does not have an inverse

20. $f(x) = 5x\sqrt{x - 1}$



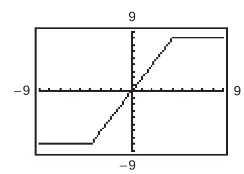
One-to-one; has an inverse

21. $g(x) = (x + 5)^3$



One-to-one; has an inverse

22. $h(x) = |x + 4| - |x - 4|$



Not one-to-one; does not have an inverse

23. $f(x) = \dfrac{x^4}{4} - 2x^2$

Not one-to-one; f does not have an inverse.

24. $f(x) = \sin\dfrac{3x}{2}$

Not one-to-one; f does not have an inverse.

25. $f(x) = 2 - x - x^3$

One-to-one; has an inverse

26. $f(x) = \sqrt[3]{x + 1}$

One-to-one; has an inverse

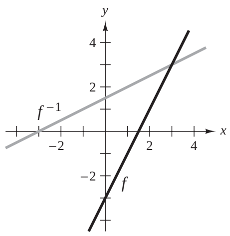
27. (a) $f(x) = 2x - 3 = y$

$$x = \frac{y+3}{2}$$

$$y = \frac{x+3}{2}$$

$$f^{-1}(x) = \frac{x+3}{2}$$

(b)



(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: all real numbers
Range of f: all real numbers
Domain of f^{-1}: all real numbers
Range of f^{-1}: all real numbers

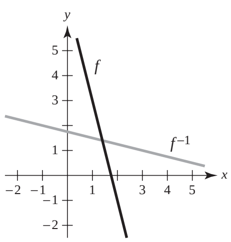
28. (a) $f(x) = 7 - 4x = y$

$$x = \frac{7-y}{4}$$

$$y = \frac{7-x}{4}$$

$$f^{-1}(x) = \frac{7-x}{4}$$

(b)



(c) The graphs of f and f^{-1} are reflections of each other across the line $y = x$.

(d) Domain of f: all real numbers
Range of f: all real numbers
Domain of f^{-1}: all real numbers
Range of f^{-1}: all real numbers

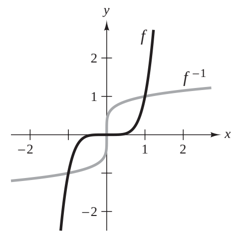
29. (a) $f(x) = x^5 = y$

$$x = \sqrt[5]{y}$$

$$y = \sqrt[5]{x}$$

$$f^{-1}(x) = \sqrt[5]{x} = x^{1/5}$$

(b)



(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: all real numbers
Range of f: all real numbers
Domain of f^{-1}: all real numbers
Range of f^{-1}: all real numbers

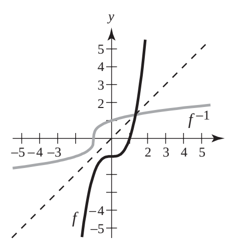
30. (a) $f(x) = x^3 - 1 = y$

$$x = \sqrt[3]{y+1}$$

$$y = \sqrt[3]{x+1}$$

$$f^{-1}(x) = \sqrt[3]{x+1} = (x+1)^{1/3}$$

(b)



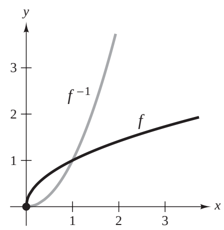
(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: all real numbers
Range of f: all real numbers
Domain of f^{-1}: all real numbers
Range of f^{-1}: all real numbers

31. (a) $f(x) = \sqrt{x} = y$

$$x = y^2$$
$$y = x^2$$
$$f^{-1}(x) = x^2, \quad x \geq 0$$

(b)



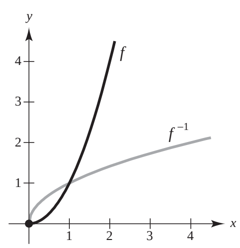
(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: $x \geq 0$
Range of f: $y \geq 0$
Domain of f^{-1}: $x \geq 0$
Range of f^{-1}: $y \geq 0$

32. (a) $f(x) = x^2 = y, \quad x \geq 0$

$$x = \sqrt{y}$$
$$y = \sqrt{x}$$
$$f^{-1}(x) = \sqrt{x}$$

(b)



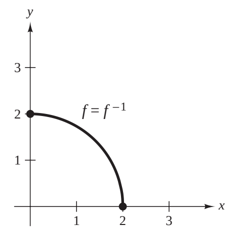
(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: $x \geq 0$
Range of f: $y \geq 0$
Domain of f^{-1}: $x \geq 0$
Range of f^{-1}: $y \geq 0$

33. (a) $f(x) = \sqrt{4 - x^2} = y, \quad 0 \leq x \leq 2$

$$4 - x^2 = y^2$$
$$x^2 = 4 - y^2$$
$$x = \sqrt{4 - y^2}$$
$$y = \sqrt{4 - x^2}$$
$$f^{-1}(x) = \sqrt{4 - x^2}, \quad 0 \leq x \leq 2$$

(b)



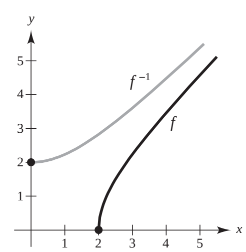
(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$. In fact, the graphs are identical.

(d) Domain of f: $0 \leq x \leq 2$
Range of f: $0 \leq y \leq 2$
Domain of f^{-1}: $0 \leq x \leq 2$
Range of f^{-1}: $0 \leq y \leq 2$

34. (a) $f(x) = \sqrt{x^2 - 4} = y, \quad x \geq 2$

$$x^2 = y^2 + 4$$
$$x = \sqrt{y^2 + 4}$$
$$y = \sqrt{x^2 + 4}$$
$$f^{-1}(x) = \sqrt{x^2 + 4}, \quad x \geq 0$$

(b)



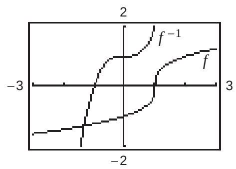
(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: $x \geq 2$
Range of f: $y \geq 0$
Domain of f^{-1}: $x \geq 0$
Range of f^{-1}: $y \geq 2$

35. (a) $f(x) = \sqrt[3]{x-1} = y$

$$x - 1 = y^3$$
$$x = y^3 + 1$$
$$y = x^3 + 1$$
$$f^{-1}(x) = x^3 + 1$$

(b)



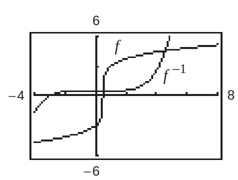
(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: all real numbers
Range of f: all real numbers
Domain of f^{-1}: all real numbers
Range of f^{-1}: all real numbers

36. (a) $f(x) = 3\sqrt[5]{2x-1} = y$

$$2x - 1 = \left(\frac{y}{3}\right)^5 = \frac{y^5}{243}$$
$$2x = \frac{y^5 + 243}{243}$$
$$x = \frac{y^5 + 243}{486}$$
$$y = \frac{x^5 + 243}{486}$$
$$f^{-1}(x) = \frac{x^5 + 243}{486}$$

(b)



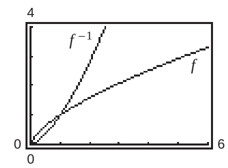
(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: all real numbers
Range of f: all real numbers
Domain of f^{-1}: all real numbers
Range of f^{-1}: all real numbers

37. (a) $f(x) = x^{2/3} = y, \quad x \ge 0$

$$x = y^{3/2}$$
$$y = x^{3/2}$$
$$f^{-1}(x) = x^{3/2}, \quad x \ge 0$$

(b)



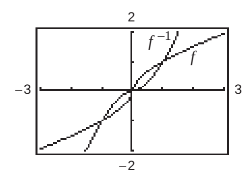
(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: $x \ge 0$
Range of f: $y \ge 0$
Domain of f^{-1}: $x \ge 0$
Range of f^{-1}: $y \ge 0$

38. (a) $f(x) = x^{3/5} = y$

$$x = y^{5/3}$$
$$y = x^{5/3}$$
$$f^{-1}(x) = x^{5/3}$$

(b)



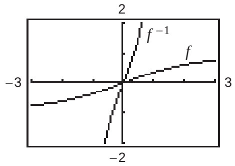
(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: all real numbers
Range of f: all real numbers
Domain of f^{-1}: all real numbers
Range of f^{-1}: all real numbers

39. (a) $f(x) = \dfrac{x}{\sqrt{x^2 + 7}} = y$

$$x = y\sqrt{x^2 + 7}$$
$$x^2 = y^2(x^2 + 7) = y^2x^2 + 7y^2$$
$$x^2(1 - y^2) = 7y^2$$
$$x = \frac{\sqrt{7}y}{\sqrt{1 - y^2}}$$
$$y = \frac{\sqrt{7}x}{\sqrt{1 - x^2}}$$
$$f^{-1}(x) = \frac{\sqrt{7}x}{\sqrt{1 - x^2}}, \quad -1 < x < 1$$

(b)



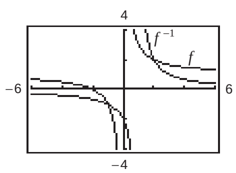
(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: all real numbers
Range of f: $-1 < y < 1$
Domain of f^{-1}: $-1 < x < 1$
Range of f^{-1}: all real numbers

40. (a)

$$f(x) = \frac{x + 2}{x} = y, \quad x \neq 0$$
$$x + 2 = yx$$
$$x(1 - y) = -2$$
$$x = \frac{2}{y - 1}$$
$$y = \frac{2}{x - 1}$$
$$f^{-1}(x) = \frac{2}{x - 1}, \quad x \neq 1$$

(b)



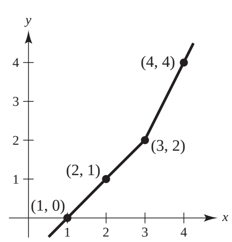
(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: all $x \neq 0$
Range of f: all $y \neq 1$
Domain of f^{-1}: all $x \neq 1$
Range of f^{-1}: all $y \neq 0$

41.

x	0	1	2	3
$f(x)$	1	2	3	4

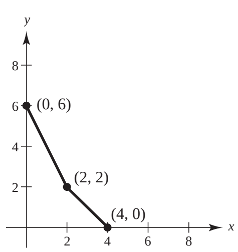
x	1	2	3	4
$f^{-1}(x)$	0	1	2	4



42.

x	0	2	6
$f(x)$	4	2	0

x	0	2	4
$f^{-1}(x)$	6	2	0



43. (a) Let x be the number of pounds of the commodity costing 1.25 per pound. Because there are 50 pounds total, the amount of the second commodity is $50 - x$. The total cost is

$$y = 1.25x + 1.60(50 - x)$$
$$= -0.35x + 80, \quad 0 \leq x \leq 50.$$

(b) Find the inverse of the original function.

$$y = -0.35x + 80$$
$$0.35x = 80 - y$$
$$x = \tfrac{100}{35}(80 - y)$$

Inverse: $y = \frac{100}{35}(80 - x) = \frac{20}{7}(80 - x)$

x represents cost and y represents pounds.

(c) Domain of inverse is $62.5 \leq x \leq 80$.

The total cost will be between \$62.50 and \$80.00.

(d) If $x = 73$ in the inverse function,

$y = \frac{100}{35}(80 - 73) = \frac{100}{5} = 20$ pounds.

44. $C = \frac{5}{9}(F - 32), \quad F \geq -459.6$

(a) $\frac{9}{5}C = F - 32$

$F = 32 + \frac{9}{5}C$

(b) The inverse function gives the Fahrenheit temperature F corresponding to the Celsius temperature C.

(c) For $F \geq -459.6$, $C = \frac{5}{9}(F - 32) \geq -273.1\overline{1}$.

So, the domain is $C \geq -273.\overline{1} = -273\frac{1}{9}$.

(d) If $C = 22°$, then $F = 32 + \frac{9}{5}(22) = 71.6°$F.

45. $f(x) = \sqrt{x - 2}, \ x \geq 2$

f is one-to-one; has an inverse.

$$y = \sqrt{x - 2}, \ x \geq 2, \ y \geq 0$$
$$y^2 = x - 2$$
$$x = y^2 + 2$$
$$f^{-1}(x) = x^2 + 2, \ x \geq 0$$

46. $f(x) = \sqrt{9 - x^2}$ is not one-to-one.

For example, $f(3) = f(-3) = 0$.

47. $f(x) = -3$

Not one-to-one; does not have an inverse.

48. $f(x) = |x - 2|, \ x \le 2$
$= -(x - 2)$
$= 2 - x$

f is one-to-one; has an inverse.

$2 - x = y$
$2 - y = x$
$f^{-1}(x) = 2 - x, \quad x \ge 0$

49. $f(x) = ax + b$

f is one-to-one; has an inverse.

$ax + b = y$
$x = \dfrac{y - b}{a}$
$y = \dfrac{x - b}{a}$
$f^{-1}(x) = \dfrac{x - b}{a}, \quad a \ne 0$

50. $f(x) = (x + a)^3 + b$

f is one-to-one; has an inverse.

$y = (x + a)^3 + b$
$y - b = (x + a)^3$
$x + a = \sqrt[3]{y - b}$
$x = \sqrt[3]{y - b} - a$
$f^{-1}(x) = \sqrt[3]{x - b} - a$

51. $f(x) = (x - 4)^2$ on $[4, \infty)$

f passes the Horizontal Line Test on $[4, \infty)$, so it is one-to-one.

52. $f(x) = |x + 2|$ on $[-2, \infty)$

f passes the Horizontal Line Test on $[-2, \infty)$, so it is one-to-one.

53. $f(x) = \dfrac{4}{x^2}$ on $(0, \infty)$

f passes the Horizontal Line Test on $(0, \infty)$, so it is one-to-one.

54. $f(x) = \cot x$ on $(0, \pi)$

f passes the Horizontal Line Test on $(0, \pi)$, so it is one-to-one.

55. $f(x) = \cos x$ on $[0, \pi]$

f passes the Horizontal Line Test on $[0, \pi]$, so it is one-to-one.

56. $f(x) = \sec x$ on $\left[0, \dfrac{\pi}{2}\right)$

f passes the Horizontal Line Test on $[0, \pi/2)$, so it is one-to-one.

57. $f(x) = (x - 3)^2$ is one-to-one for $x \ge 3$.

$(x - 3)^2 = y$
$x - 3 = \sqrt{y}$
$x = \sqrt{y} + 3$
$y = \sqrt{x} + 3$
$f^{-1}(x) = \sqrt{x} + 3, \quad x \ge 0$

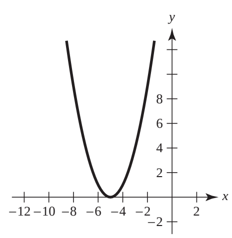
(Answer is not unique.)

58. $f(x) = |x - 3|$ is one-to-one for $x \ge 3$.

$x - 3 = y$
$x = y + 3$
$y = x + 3$
$f^{-1}(x) = x + 3, \quad x \ge 0$

(Answer is not unique.)

59. (a) $f(x) = (x + 5)^2$

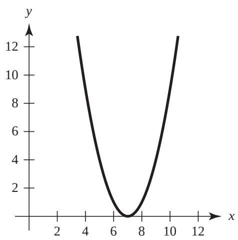


(b) f is one-to-one on $[-5, \infty)$. (Note that f is also one-to-one on $(-\infty, -5]$.)

(c) $f(x) = (x + 5)^2 = y, \qquad x \ge -5$
$x + 5 = \sqrt{y}$
$x = \sqrt{y} - 5$
$y = \sqrt{x} - 5$
$f^{-1}(x) = \sqrt{x} - 5$

(d) Domain of f^{-1}: $x \ge 0$

60. (a) $f(x) = (7 - x)^2 = (x - 7)^2$

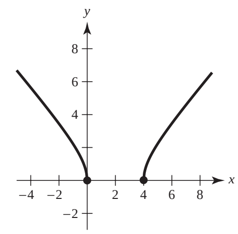


(b) f is one-to-one on $[7, \infty)$. (Note that f is also one-to-one on $(-\infty, 7]$.)

(c)
$$\begin{aligned} f(x) = (x - 7)^2 &= y, \quad x \geq 7 \\ x - 7 &= \sqrt{y} \\ x &= 7 + \sqrt{y} \\ y &= 7 + \sqrt{x} \\ f^{-1}(x) &= 7 + \sqrt{x} \end{aligned}$$

(d) Domain of f^{-1}: $x \geq 0$

61. (a) $f(x) = \sqrt{x^2 - 4x}$

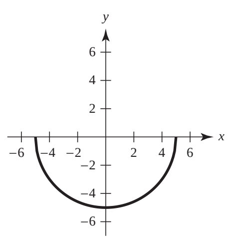


(b) f is one-to-one on $[4, \infty)$. (Note that f is also one-to-one on $(-\infty, 0]$.)

(c)
$$\begin{aligned} f(x) = \sqrt{x^2 - 4x} &= y, \quad x \geq 4 \\ x^2 - 4x &= y^2 \\ x^2 - 4x + 4 &= y^2 + 4 \\ (x - 2)^2 &= y^2 + 4 \\ x - 2 &= \sqrt{y^2 + 4} \\ x &= 2 + \sqrt{y^2 + 4} \\ y &= 2 + \sqrt{x^2 + 4} \\ f^{-1}(x) &= 2 + \sqrt{x^2 + 4} \end{aligned}$$

(d) Domain of f^{-1}: $x \geq 0$

62. (a) $f(x) = -\sqrt{25 - x^2}$

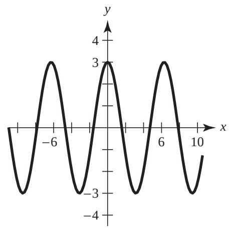


(b) f is one-to-one on $[0, 5]$. (Note that f is also one-to-one on $[-5, 0]$.)

(c)
$$\begin{aligned} f(x) = -\sqrt{25 - x^2} &= y, \quad 0 \leq x \leq 5, -5 \leq y \leq 0 \\ 25 - x^2 &= y^2 \\ x^2 &= 25 - y^2 \\ x &= \sqrt{25 - y^2} \\ y &= \sqrt{25 - x^2} \\ f^{-1}(x) &= \sqrt{25 - x^2} \end{aligned}$$

(d) Domain of f^{-1}: $-5 \leq x \leq 0$

63. (a) $f(x) = 3\cos x$



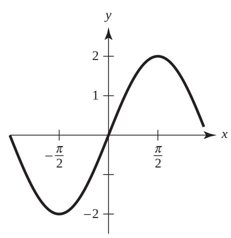
(b) f is one-to-one on $[0, \pi]$. (other answers possible)

(c) $f(x) = 3\cos x = y$

$$\cos x = \frac{y}{3}$$
$$x = \arccos\left(\frac{y}{3}\right)$$
$$y = \arccos\left(\frac{x}{3}\right)$$
$$f^{-1}(x) = \arccos\left(\frac{x}{3}\right)$$

(d) Domain of f^{-1}: $-3 \le x \le 3$

64. (a) $f(x) = 2\sin x$



(b) f is one-to-one on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. (other answers possible)

(c) $f(x) = 2\sin x = y$

$$\sin x = \frac{y}{2}$$
$$x = \arcsin\left(\frac{y}{2}\right)$$
$$y = \arcsin\left(\frac{x}{2}\right)$$
$$f^{-1}(x) = \arcsin\left(\frac{x}{2}\right)$$

(d) Domain of f^{-1}: $-2 \le x \le 2$

65. $f(x) = x^3 + 2x - 1$

$f(1) = 2 = a \Rightarrow f^{-1}(2) = 1$

66. $f(x) = 2x^5 + x^3 + 1$

$f(-1) = -2 = a \Rightarrow f^{-1}(-2) = -1$

67. $f(x) = \sin x$

$f\left(\frac{\pi}{6}\right) = \frac{1}{2} = a \Rightarrow f^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

68. $f(x) = \cos 2x$

$f(0) = 1 = a \Rightarrow f^{-1}(1) = 0$

69. $f(x) = x^3 - \frac{4}{x}$

$f(2) = 6 = a \Rightarrow f^{-1}(6) = 2$

70. $f(x) = \sqrt{x - 4}$

$f(8) = 2 = a \Rightarrow f^{-1}(2) = 8$

In Exercises 71–74, use the following.

$f(x) = \frac{1}{8}x - 3$ **and** $g(x) = x^3$

$f^{-1}(x) = 8(x + 3)$ **and** $g^{-1}(x) = \sqrt[3]{x}$

71. $(f^{-1} \circ g^{-1})(1) = f^{-1}(g^{-1}(1)) = f^{-1}(1) = 32$

72. $(g^{-1} \circ f^{-1})(-3) = g^{-1}(f^{-1}(-3)) = g^{-1}(0) = 0$

73. $(f^{-1} \circ f^{-1})(6) = f^{-1}(f^{-1}(6)) = f^{-1}(72) = 600$

74. $(g^{-1} \circ g^{-1})(-4) = g^{-1}(g^{-1}(-4)) = g^{-1}(\sqrt[3]{-4})$

$$= \sqrt[3]{\sqrt[3]{-4}} = -\sqrt[9]{4}$$

In Exercises 75–78, use the following.

$f(x) = x + 4$ **and** $g(x) = 2x - 5$

$f^{-1}(x) = x - 4$ **and** $g^{-1}(x) = \frac{x + 5}{2}$

75.
$$\begin{aligned}(g^{-1} \circ f^{-1})(x) &= g^{-1}(f^{-1}(x)) \\ &= g^{-1}(x - 4) \\ &= \frac{(x - 4) + 5}{2} \\ &= \frac{x + 1}{2}\end{aligned}$$

76.
$$\begin{aligned}(f^{-1} \circ g^{-1})(x) &= f^{-1}(g^{-1}(x)) \\ &= f^{-1}\left(\frac{x + 5}{2}\right) \\ &= \frac{x + 5}{2} - 4 \\ &= \frac{x - 3}{2}\end{aligned}$$

77. $(f \circ g)(x) = f(g(x))$

$= f(2x - 5)$

$= (2x - 5) + 4$

$= 2x - 1$

So, $(f \circ g)^{-1}(x) = \dfrac{x + 1}{2}$.

Note: $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$

78. $(g \circ f)(x) = g(f(x))$

$= g(x + 4)$

$= 2(x + 4) - 5$

$= 2x + 3$

So, $(g \circ f)^{-1}(x) = \dfrac{x - 3}{2}$.

Note: $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$

79. (a) f is one-to-one because it passes the Horizontal Line Test.

(b) The domain of f^{-1} is the range of f: $[-2, 2]$.

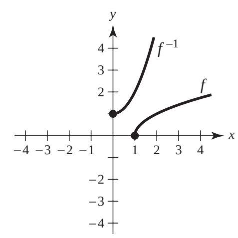
(c) $f^{-1}(2) = -4$ because $f(-4) = 2$.

80. (a) f is one-to-one because it passes the Horizontal Line Test.

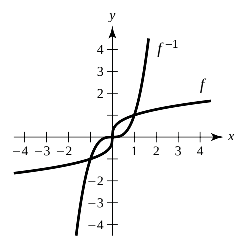
(b) The domain of f^{-1} is the range of f: $[-3, 3]$.

(c) $f^{-1}(2) \approx 1.73$ because $f(1.73) \approx 2$.

81.



82.

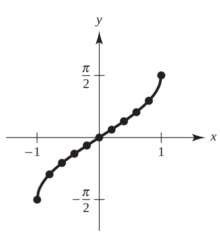


83. $y = \arcsin x$

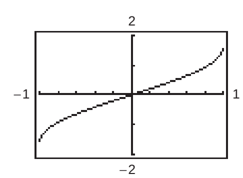
(a)

x	−1	−0.8	−0.6	−0.4	−0.2	0	0.2	0.4	0.6	0.8	1
y	−1.571	−0.927	−0.644	−0.412	−0.201	0	0.201	0.412	0.644	0.927	1.571

(b)



(c)



(d) Symmetric about origin: $\arcsin(-x) = -\arcsin x$

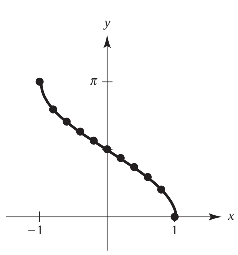
Intercept: $(0, 0)$

84. $y = \arccos x$

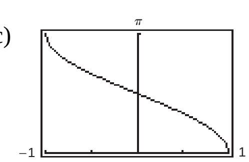
(a)

x	−1	−0.8	−0.6	−0.4	−0.2	0	0.2	0.4	0.6	0.8	1
y	3.142	2.498	2.214	1.982	1.772	1.571	1.369	1.159	0.927	0.644	0

(b)



(c)



(d) Intercepts: $\left(0, \dfrac{\pi}{2}\right)$ and $(1, 0)$

85. $y = \arccos x$

$\left(-\frac{\sqrt{2}}{2}, \frac{3\pi}{4}\right)$ because $\cos\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$.

$\left(\frac{1}{2}, \frac{\pi}{3}\right)$ because $\cos\left(\frac{\pi}{4}\right) = \frac{1}{2}$.

$\left(\frac{\sqrt{3}}{2}, \frac{\pi}{6}\right)$ because $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$.

86. No, g is not the inverse of f. $f(x) = \sin x$ is not one-to-one. The graph of g is not the graph of a function.

87. $\arcsin \frac{1}{2} = \frac{\pi}{6}$

88. $\arcsin 0 = 0$

89. $\arccos \frac{1}{2} = \frac{\pi}{3}$

90. $\arccos 1 = 0$

91. $\arctan \frac{\sqrt{3}}{3} = \frac{\pi}{6}$

92. $\text{arccot}\left(-\sqrt{3}\right) = \frac{5\pi}{6}$

93. $\text{arccsc}\left(-\sqrt{2}\right) = -\frac{\pi}{4}$

94. $\text{arcsec}\left(-\sqrt{2}\right) = \frac{3\pi}{4}$

95. $\arccos(0.8) \approx 2.50$

96. $\arcsin(-0.39) \approx -0.40$

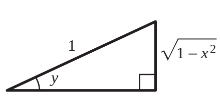
97. $\text{arcsec}(1.269) = \arccos\left(\frac{1}{1.269}\right) \approx 0.66$

98. $\arctan(-5) \approx -1.37$

99. $\cos\left[\arccos(-0.1)\right] = -0.1$

100. $\arcsin(\sin 3\pi) = \arcsin(0) = 0$

In Exercises 101–106, use the triangle.



101. $y = \arccos x$

$\cos y = x$

102. $\sin y = \sqrt{1 - x^2}$

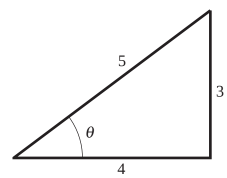
103. $\tan y = \frac{\sqrt{1 - x^2}}{x}$

104. $\cot y = \frac{x}{\sqrt{1 - x^2}}$

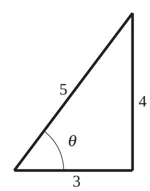
105. $\sec y = \frac{1}{x}$

106. $\csc y = \frac{1}{\sqrt{1 - x^2}}$

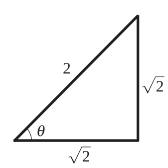
107. (a) $\sin\left(\arctan\frac{3}{4}\right) = \frac{3}{5}$



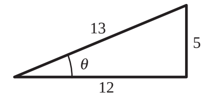
(b) $\sec\left(\arcsin\frac{4}{5}\right) = \frac{5}{3}$



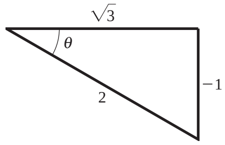
108. (a) $\tan\left(\arccos\frac{\sqrt{2}}{2}\right) = \tan\left(\frac{\pi}{4}\right) = 1$



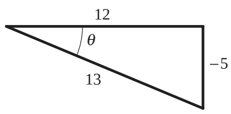
(b) $\cos\left(\arcsin\frac{5}{13}\right) = \frac{12}{13}$



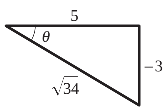
109. (a) $\cot\left[\arcsin\left(-\frac{1}{2}\right)\right] = \cot\left(-\frac{\pi}{6}\right) = -\sqrt{3}$



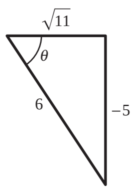
(b) $\csc\left[\arctan\left(-\frac{5}{12}\right)\right] = -\frac{13}{5}$



110. (a) $\sec\left[\arctan\left(-\frac{3}{5}\right)\right] = \frac{\sqrt{34}}{5}$



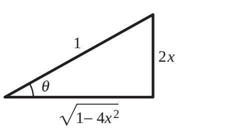
(b) $\tan\left[\arcsin\left(-\frac{5}{6}\right)\right] = -\frac{5\sqrt{11}}{11}$



111. $y = \cos(\arcsin 2x)$

$\theta = \arcsin 2x$

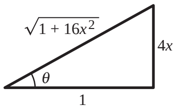
$y = \cos\theta = \sqrt{1 - 4x^2}$



112. $\sec(\arctan 4x)$

$\theta = \arctan 4x$

$y = \sec\theta = \sqrt{16x^2 + 1}$

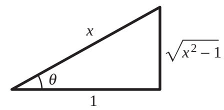


113. $y = \sin(\operatorname{arcsec} x)$

$\theta = \operatorname{arcsec} x,\ 0 \le \theta \le \pi,\ \theta \ne \frac{\pi}{2}$

$y = \sin\theta = \frac{\sqrt{x^2 - 1}}{|x|}$

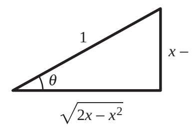
The absolute value bars on x are necessary because of the restriction $0 \le \theta \le \pi$, $\theta \ne \pi/2$, and $\sin\theta$ for this domain must always be nonnegative.



114. $y = \sec[\arcsin(x - 1)]$

$\theta = \arcsin(x - 1)$

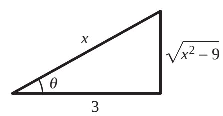
$y = \sec\theta = \frac{1}{\sqrt{2x - x^2}}$



115. $y = \tan\left(\operatorname{arcsec}\frac{x}{3}\right)$

$\theta = \operatorname{arcsec}\frac{x}{3}$

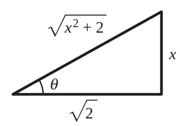
$y = \tan\theta = \frac{x^2 - 9}{3}$



116. $y = \csc\left(\arctan\frac{x}{\sqrt{2}}\right)$

$\theta = \arctan\frac{x}{\sqrt{2}}$

$y = \csc\theta = \frac{\sqrt{x^2 + 2}}{x}$



117. $\arcsin(3x - \pi) = \frac{1}{2}$

$3x - \pi = \sin\left(\frac{1}{2}\right)$

$x = \frac{1}{3}\left[\sin\left(\frac{1}{2}\right) + \pi\right] \approx 1.207$

118. $\arctan(2x - 5) = -1$

$2x - 5 = \tan(-1)$

$x = \frac{1}{2}\left(5 + \tan(-1)\right) \approx 1.721$

119. $\arcsin \sqrt{2x} = \arccos \sqrt{x}$

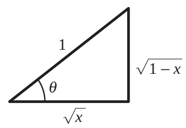
$$\sqrt{2x} = \sin\left(\arccos \sqrt{x}\right)$$

$$\sqrt{2x} = \sqrt{1 - x},\ 0 \le x \le 1$$

$$2x = 1 - x$$

$$3x = 1$$

$$x = \tfrac{1}{3}$$



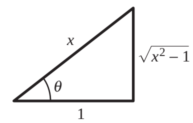
120. $\arccos x = \operatorname{arcsec} x$

$$x = \cos\left(\operatorname{arcsec} x\right)$$

$$x = \frac{1}{x}$$

$$x^2 = 1$$

$$x = \pm 1$$



121. $y = \arccos x$

$y = \arctan x$

The point of intersection is given by
$f(x) = \arccos x - \arctan x = 0,\ \cos(\arccos x) = \cos(\arctan x).$

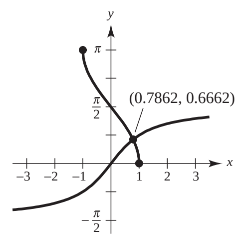
$$x = \frac{1}{\sqrt{1 + x^2}}$$

$$x^2\left(1 + x^2\right) = 1$$

$$x^4 + x^2 - 1 = 0 \text{ when } x^2 = \frac{-1 + \sqrt{5}}{2}.$$

So, $x = \pm\sqrt{\dfrac{-1 + \sqrt{5}}{2}} \approx \pm 0.7862.$

Point of intersection: $(0.7862,\ 0.6662)$ $\left[\text{Because } f(-0.7862) = \pi \ne 0.\right]$



122. $y = \arcsin x$

$y = \arccos x$

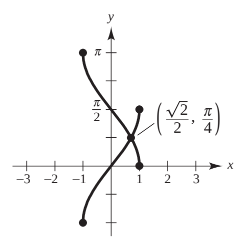
The point of intersection is given by
$f(x) = \arcsin x - \arccos x = 0,\ \sin(\arcsin x) = \sin(\arccos x).$

$$x = 1 - x^2$$

$$x^2 = 1 - x^2$$

$$x = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$$

Point of intersection: $\left(\sqrt{2}/2,\ \pi/4\right)$ $\left[\text{Because } f\left(-\sqrt{2}/2\right) = -\pi \ne 0.\right]$



123. Let $y = f(x)$ be one-to-one. Solve for x as a function of y. Interchange x and y to get $y = f^{-1}(x)$. Let the domain of f^{-1} be the range of f. Verify that $f\left(f^{-1}(x)\right) = x$ and $f^{-1}\left(f(x)\right) = x$.

Example:

$$f(x) = x^3$$

$$y = x^3$$

$$x = \sqrt[3]{y}$$

$$y = \sqrt[3]{x}$$

$$f^{-1}(x) = \sqrt[3]{x}$$

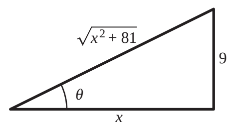
124. The graphs of f and f^{-1} are mirror images with respect to the line $y = x$.

125. The trigonometric functions are not one-to-one. So, their domains must be restricted to define the inverse trigonometric functions.

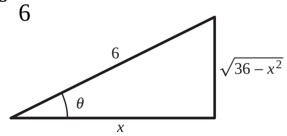
126. You could graph $f(x) = \operatorname{arccot}(x)$ as follows.

$$f(x) = \begin{cases} \arctan(1/x) + \pi, & -\infty < x < 0 \\ \pi/2, & x = 0 \\ \arctan(1/x), & 0 < x < \infty \end{cases}$$

127. $\arctan \dfrac{9}{x} = \arcsin \dfrac{9}{\sqrt{x^2 + 81}}$



128. $\arcsin \dfrac{\sqrt{36 - x^2}}{6} = \arccos \dfrac{x}{6}$



129. (a) $\operatorname{arccsc} x = \arcsin \dfrac{1}{x}, \ |x| \geq 1$

Let $y = \operatorname{arccsc} x$.

Then for $-\dfrac{\pi}{2} \leq y < 0$ and $0 < y \leq \dfrac{\pi}{2}$,

$\csc y = x \Rightarrow \sin y = \dfrac{1}{x}$.

So, $y = \arcsin\left(\dfrac{1}{x}\right)$. Therefore,

$\operatorname{arccsc} x = \arcsin\left(\dfrac{1}{x}\right)$.

(b) $\arctan x + \arctan\dfrac{1}{x} = \dfrac{\pi}{2}, \ x > 0$

Let $y = \arctan x + \arctan(1/x)$.

$$\text{Then } \tan y = \frac{\tan(\arctan x) + \tan[\arctan(1/x)]}{1 - \tan(\arctan x)\tan[\arctan(1/x)]}$$
$$= \frac{x + (1/x)}{1 - x(1/x)}$$
$$= \frac{x + (1/x)}{0} \text{ (which is undefined).}$$

So, $y = \pi/2$. Therefore,
$\arctan x + \arctan(1/x) = \pi/2$.

130. (a) $\arcsin(-x) = -\arcsin x, \ |x| \leq 1$

Let $y = \arcsin(-x)$.

Then $-x = \sin y \Rightarrow x = -\sin y \Rightarrow x = \sin(-y)$.

So, $-y = \arcsin x \Rightarrow y = -\arcsin x$.

Therefore, $\arcsin(-x) = -\arcsin x$.

(b) $\arccos(-x) = \pi - \arccos x, \ |x| \leq 1$

Let $y = \arccos(-x)$. Then
$-x = \cos y \Rightarrow x = -\cos y \Rightarrow x = \cos(\pi - y)$.

So, $\pi - y = \arccos x \Rightarrow y = \pi - \arccos x$.

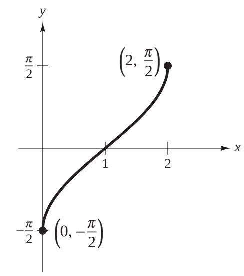
Therefore, $\arccos(-x) = \pi - \arccos x$.

131. $f(x) = \arcsin(x - 1)$
$x - 1 = \sin y$
$x = 1 + \sin y$

Domain: $[0, 2]$

Range: $[-\pi/2, \pi/2]$

$f(x)$ is the graph of arcsin x shifted right one unit.



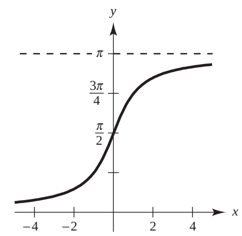
132. $f(x) = \arctan x + \dfrac{\pi}{2}$

$x = \tan\left(y - \dfrac{\pi}{2}\right)$

Domain: $(-\infty, \infty)$

Range: $[0, \pi]$

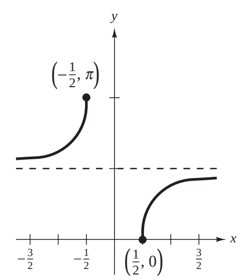
$f(x)$ is the graph of arctan x shifted $\pi/4$ unit upward.



133. $f(x) = \operatorname{arcsec} 2x$
$2x = \sec y$
$x = \dfrac{1}{2}\sec y$

Domain: $(-\infty, -1/2], [1/2, \infty)$

Range: $[0, \pi/2), (\pi/2, \pi]$



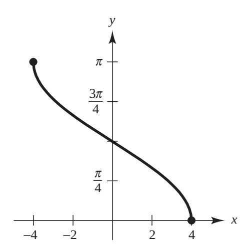
134. $f(x) = \arccos\left(\dfrac{x}{4}\right)$

$\dfrac{x}{4} = \cos y$

$x = 4 \cos y$

Domain: $[-4, 4]$

Range: $[0, \pi]$



135. Because $f(-3) = 8$ and f is one-to-one, you have $f^{-1}(8) = -3$.

136. Because $f(0) = 5 + \arccos(0) = 5 + \pi/2$, and f is one-to-one, $f^{-1}(\pi/2 + 5) = 0$.

137. Let f and g be one-to-one functions.

Let $(f \circ g)(x) = y$, then $x = (f \circ g)^{-1}(y)$. Also:

$$(f \circ g)(x) = y$$
$$f(g(x)) = y$$
$$g(x) = f^{-1}(y)$$
$$x = g^{-1}(f^{-1}(y))$$
$$x = (g^{-1} \circ f^{-1}(y))$$

So, $(f \circ g)^{-1}(y) = (g^{-1} \circ f^{-1})(y)$ and

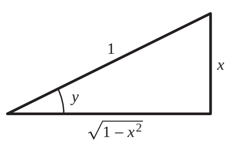
$(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

138. If f has an inverse, then f and f^{-1} are both one-to-one.

Let $(f^{-1})^{-1}(x) = y$ then $x = f^{-1}(y)$ and $f(x) = y$.

So, $(f^{-1})^{-1} = f$.

139. Let $y = \sin^{-1} x$. Then $\sin y = x$ and $\cos(\sin^{-1} x) = \cos(y) = \sqrt{1 - x^2}$, as indicated in the figure.



140. Suppose $g(x)$ and $h(x)$ are both inverses of $f(x)$. Then the graph of $f(x)$ contains the point (a, b) if and only if the graphs of $g(x)$ and $h(x)$ contain the point (b, a). Because the graphs of $g(x)$ and $h(x)$ are the same, $g(x) = h(x)$. So, the inverse of $f(x)$ is unique.

141. False. Let $f(x) = x^2$.

142. True; if f has a y-intercept.

143. False

$$\arcsin^2 0 + \arccos^2 0 = 0 + \frac{\pi}{2}^2 \neq 1$$

144. False

The range of $y = \arcsin x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

145. True

146. False. Let $f(x) = x$ or $g(x) = 1/x$.

147. (a) $\operatorname{arccot} x = y$ if and only if $\cot y = x$, $0 < y < \pi$.

For $x > 0$, $\cot y > 0$ and $0 < y < \frac{\pi}{2}$.

So, $\tan y = \frac{1}{x} > 0$ and $y = \arctan\left(\frac{1}{x}\right)$.

For $x = 0$, $\operatorname{arccot}(0) = \frac{\pi}{2}$.

For $x < 0$, $\cot y < 0$ and $\frac{\pi}{2} < y < \pi$.

So, $\tan y = \frac{1}{x} < 0$ and $\arctan\left(\frac{1}{x}\right) < 0$.

Therefore, you need to add π to get $y = \pi + \arctan\left(\frac{1}{x}\right)$.

(b) $y = \operatorname{arcsec} x$ if and only if $\sec y = x$, $|x| \geq 1$, $0 \leq y \leq \pi$, $y \neq \frac{\pi}{2}$.

So, $\cos y = \frac{1}{x}$ and $y = \arccos\left(\frac{1}{x}\right)$.

(c) $y = \operatorname{arccsc} x$ if and only if $\csc y = x$, $|x| \geq 1$, $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$, $y \neq 0$.

So, $\sin y = \frac{1}{x}$ and $y = \arcsin\left(\frac{1}{x}\right)$.

148. (a) $\operatorname{arccot}(0.5) = \arctan\left(\frac{1}{0.5}\right) = \arctan(2) \approx 1.1071$

(b) $\operatorname{arcsec}(2.7) = \arccos\left(\frac{1}{2.7}\right) \approx 1.1914$

(c) $\operatorname{arccsc}(-3.9) = \arcsin\left(\frac{-1}{3.9}\right) \approx -0.2593$

(d) $\operatorname{arccot}(-0.5) = \pi + \arctan(-2.0) \approx 2.0344$

149. $\tan(\arctan x + \arctan y) = \dfrac{\tan(\arctan x + \arctan y)}{1 - \tan(\arctan x)\tan(\arctan y)} = \dfrac{x+y}{1-xy}, xy \neq 1$

So,

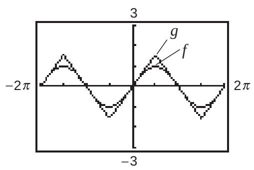
$\arctan x + \arctan y = \arctan\left(\dfrac{x+y}{1-xy}\right), xy \neq 1.$

Let $x = \dfrac{1}{2}$ and $y = \dfrac{1}{3}$.

$$\arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{3}\right) = \arctan \frac{\frac{1}{2}+\frac{1}{3}}{1-\left(\frac{1}{2}\cdot\frac{1}{3}\right)} = \arctan\frac{\frac{5}{6}}{1-\frac{1}{6}} = \arctan\frac{\frac{5}{6}}{\frac{5}{6}} = \arctan 1 = \frac{\pi}{4}$$

150. $\arcsin(\sin x) \neq x$ for many values of x outside $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$.

For example, $\arcsin(\sin 2\pi) = \arcsin(0) = 0 \neq 2\pi$.



151. $y = ax^2 + bx + c$. Interchange x and y, and solve for y using the quadratic formula.

$$ay^2 + by + c - x = 0$$

$$y = \frac{-b \pm \sqrt{b^2 - 4a(c-x)}}{2a}$$

Because $x \leq \dfrac{-b}{2a}$, use the negative sign.

$$f^{-1}(x) = \frac{-b - \sqrt{b^2 - 4ac + 4ax}}{2a}$$

153. f is one-to-one if $f(x_1) = f(x_2)$ implies $x_1 = x_2$. So assume

$$\begin{aligned} f(x_1) &= f(x_2) \\ \frac{ax_1+b}{cx_1+d} &= \frac{ax_2+b}{cx_2+d} \\ acx_1x_2 + adx_1 + bcx_2 + bd &= acx_1x_2 + adx_2 + bcx_1 + bd \\ adx_1 + bcx_2 &= adx_2 + bcx_1 \\ (ad-bc)x_1 &= (ad-bc)x_2. \end{aligned}$$

So, $x_1 = x_2$ if $ad - bc \neq 0$. To find f^{-1}, solve for x as follows.

$$\begin{aligned} y &= \frac{ax+b}{cx+d} \\ ycx + yd &= ax + b \\ (yc-a)x &= b - yd \\ x &= \frac{b-yd}{yc-a} \\ f^{-1}(x) &= \frac{b-dx}{cx-a} \end{aligned}$$

152. f will be symmetric about the line $y = x$ if f is one-to-one, and equals its inverse. So assume

$$\begin{aligned} f(x_1) &= f(x_2) \\ \frac{ax_1+b}{cx_1-a} &= \frac{ax_2+b}{cx_2-a} \\ acx_1x_2 - a^2x_1 + bcx_2 - ab &= acx_1x_2 + bcx_1 - ab \\ (a^2+bc)x_2 &= (a^2+bc)x_1. \end{aligned}$$

So, $x_1 = x_2$ if $a^2 + bc \neq 0$.

To show that $f = f^{-1}$, solve for x as follows:

$$\begin{aligned} y &= \frac{ax+b}{cx-a} \\ ycx - ay &= ax + b \\ (yc-a)x &= b + ay \\ x &= \frac{ay+b}{yc-a} \\ f^{-1}(x) &= \frac{ax+b}{cx-a} = f(x) \end{aligned}$$

So, f is symmetric about the line $y = x$ is and only if $a^2 + bc \neq 0$.

Section 1.6 Exponential and Logarithmic Functions

1. (a) $25^{3/2} = 5^3 = 125$

(b) $81^{1/2} = 9$

(c) $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

(d) $27^{-1/3} = \frac{1}{27^{1/3}} = \frac{1}{3}$

2. (a) $64^{1/3} = 4$

(b) $5^{-4} = \frac{1}{5^4} = \frac{1}{625}$

(c) $\left(\frac{1}{8}\right)^{1/3} = \frac{1}{2}$

(d) $\left(\frac{1}{4}\right)^3 = \frac{1}{64}$

3. (a) $(5^2)(5^3) = 5^{2+3} = 5^5 = 3125$

(b) $(5^2)(5^{-3}) = 5^{2-3} = 5^{-1} = \frac{1}{5}$

(c) $\frac{5^3}{25^2} = \frac{5^3}{5^4} = \frac{1}{5}$

(d) $\left(\frac{1}{4}\right)^2 2^6 - \frac{2^6}{2^4} = 2^2 = 4$

4. (a) $(2^2)^3 = 2^6 = 64$

(b) $(5^4)^{1/2} = 5^2 = 25$

(c) $\left[(27)^{-1}(27)^{2/3}\right]^3 = \left[27^{-1/3}\right]^3 = 27^{-1} = \frac{1}{27}$

(d) $(25)^{3/2}3^2 = 5^3 3^2 = (125)9 = 1125$

5. (a) $e^2(e^4) = e^6$

(b) $(e^3)^4 = e^{12}$

(c) $(e^3)^{-2} = e^{-6} = \frac{1}{e^6}$

(d) $\frac{e^5}{e^3} = e^2$

6. (a) $\left(\frac{1}{e}\right)^{-2} = e^2$

(b) $(e^3)^4 = e^{12}$

(c) $e^0 = 1$

(d) $\frac{1}{e^{-3}} = e^3$

7. $3^x = 81 \Rightarrow x = 4$

8. $4^x = 64 \Rightarrow x = 3$

9. $6^{x-2} = 36 \Rightarrow x - 2 = 2 \Rightarrow x = 4$

10. $5^{x+1} = 125 \Rightarrow x + 1 = 3 \Rightarrow x = 2$

11. $\left(\frac{1}{2}\right)^x = 32 \Rightarrow 2^{-x} = 32 \Rightarrow -x = 5 \Rightarrow x = -5$

12. $\left(\frac{1}{4}\right)^x = 16 \Rightarrow 4^{-x} = 16 \Rightarrow -x = 2 \Rightarrow x = -2$

13. $\left(\frac{1}{3}\right)^{x-1} = 27 \Rightarrow 3^{1-x} = 27 \Rightarrow 1 - x = 3 \Rightarrow x = -2$

14. $\left(\frac{1}{5}\right)^{2x} = 625 \Rightarrow 5^{-2x} = 5^4 \Rightarrow -2x = 4 \Rightarrow x = -2$

15. $4^3 = (x + 2)^3 \Rightarrow 4 = x + 2 \Rightarrow x = 2$

16. $18^2 = (5x - 7)^2 \Rightarrow \pm 18 = 5x - 7 \Rightarrow x = 5, \ -\frac{11}{5}$

17. $x^{3/4} = 8 \Rightarrow x = 8^{4/3} = 2^4 = 16$

18. $(x + 3)^{4/3} = 16 \Rightarrow x + 3 = \pm 16^{3/4}$

$\Rightarrow x + 3 = \pm 8 \Rightarrow x = 5, -11$

19. $e^x = 5 \Rightarrow x = \ln 5 \approx 1.609$

20. $e^x = 1 = e^0 \Rightarrow x = 0$

21. $e^{-2x} = e^5 \Rightarrow -2x = 5 \Rightarrow x = -\frac{5}{2}$

22. $e^{3x} = e^{-4} \Rightarrow 3x = -4 \Rightarrow x = -\frac{4}{3}$

23. $\left(1 + \frac{1}{1{,}000{,}000}\right)^{1{,}000{,}000} \approx 2.718280469$

$e \approx 2.718281828$

$e > \left(1 + \frac{1}{1{,}000{,}000}\right)^{1{,}000{,}000}$

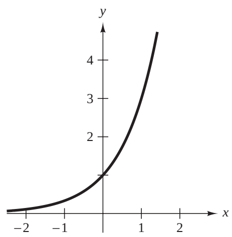
24. $1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} + \frac{1}{720} + \frac{1}{5040} = 2.71\overline{825396}$

$$e \approx 2.718281828$$

$$e > 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} + \frac{1}{720} + \frac{1}{5040}$$

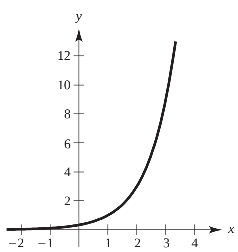
25. $y = 3^x$

x	-2	-1	0	1	2
y	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9



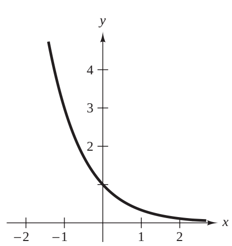
26. $y = 3^{x-1}$

x	-1	0	1	2	3
y	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9



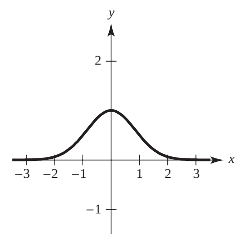
27. $y = \left(\frac{1}{3}\right)^x = 3^{-x}$

x	-2	-1	0	1	2
y	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$



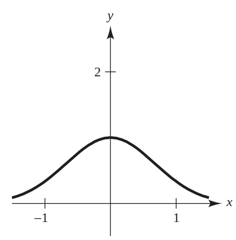
28. $y = 2^{-x^2}$

x	-2	-1	0	1	2	3
y	$\frac{1}{16}$	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{16}$	0.002



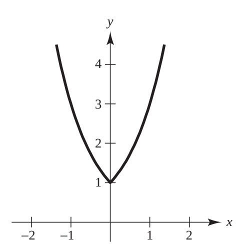
29. $f(x) = 3^{-x^2}$

x	0	± 1	± 2
y	1	$\frac{1}{3}$	0.0123



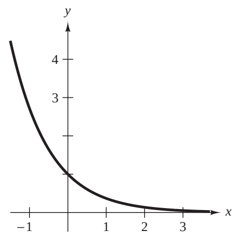
30. $f(x) = 3^{|x|}$

x	0	± 1	± 2
y	1	9	9



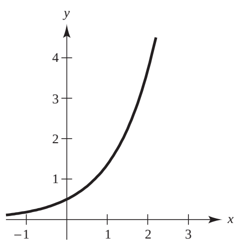
31. $y = e^{-x}$

x	-1	0	1
y	e	1	$\frac{1}{e}$



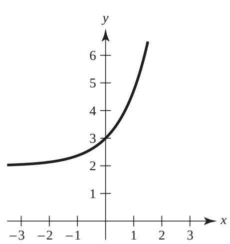
32. $y = \frac{1}{2}e^{x}$

x	-1	0	1	2
y	$\frac{1}{2e}$	$\frac{1}{2}$	$\frac{e}{2}$	$\frac{e^2}{2}$



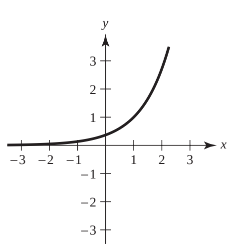
33. $y = e^{x} + 2$

x	-2	-1	0	1	2
y	$\frac{1}{e^2} + 2$	$\frac{1}{e} + 2$	3	$e + 2$	$e^2 + 2$



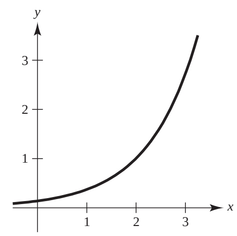
34. $y = e^{x-1}$

x	-1	0	1	2
y	$\frac{1}{e^2}$	$\frac{1}{e}$	1	e



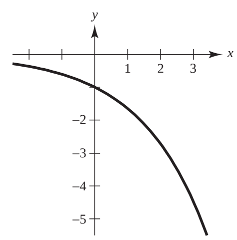
35. $h(x) = e^{x-2}$

x	0	1	2	3	4
y	e^{-2}	e^{-1}	1	e	e^2



36. $g(x) = -e^{x/2}$

x	-2	0	2	4
y	$-\frac{1}{e}$	-1	$-e$	$-e^2$

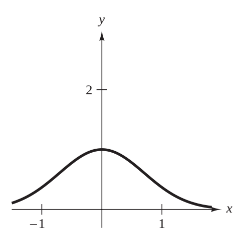


37. $y = e^{-x^2}$

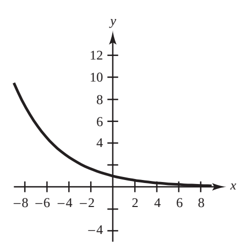
Symmetric with respect to the y-axis

Horizontal asymptote

$y = 0$



38. $y = e^{-x/4}$



39. $f(x) = \dfrac{1}{3 + e^x}$

Because $e^x > 0,\ 3 + e^x > 0.$

Domain: all real numbers

40. $f(x) = \dfrac{1}{2 - e^x}$

$2 - e^x = 0 \Rightarrow x = \ln 2$

Domain: all $x \neq \ln 2$

41. $f(x) = \sqrt{1 - 4^x}$

$1 - 4^x \geq 0 \Rightarrow 4^x \leq 1 \Rightarrow x \ln 4 \leq \ln 1 = 0$

Domain: $x \leq 0$

42. $f(x) = \sqrt{1 + 3^{-x}}$

Because $1 + 3^{-x} > 0$ for all x, the domain is all real numbers.

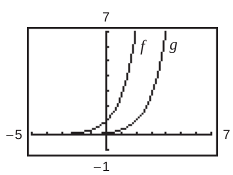
43. $f(x) = \sin e^{-x}$

Domain: all real numbers

44. $f(x) = \cos e^{-x}$

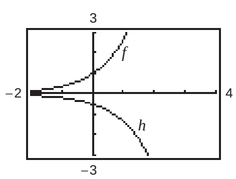
Domain: all real numbers

45. (a)



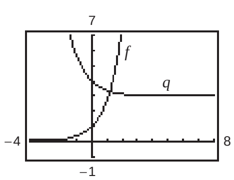
Horizontal shift 2 units to the right.

(b)



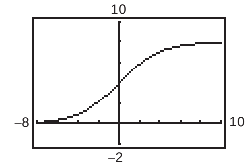
A reflection in the x-axis and a vertical shrink.

(c)



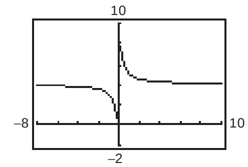
Vertical shift 3 units upward and a reflection in the y-axis.

46. (a)



The graph approaches 8 as $x \to \infty$. The graph approaches 0 as $x \to -\infty$.

(b)



As $x \to \pm\infty$, the graph approaches 4.

47. $y = Ce^{ax}$

Horizontal asymptote: $y = 0$

Matches (c)

48. $y = Ce^{-ax}$

Horizontal asymptote: $y = 0$

Reflection in the y-axis

Matches (d)

49. $y = C(1 - e^{-ax})$

Vertical shift C units

Reflection in both the x- and y-axes

Matches (a)

50. $y = \dfrac{C}{1 + e^{-ax}}$

$\lim_{x \to \infty} \dfrac{C}{1 + e^{-ax}} = C$

$\lim_{x \to -\infty} \dfrac{C}{1 + e^{-ax}} = 0$

Horizontal asymptotes: $y = C$ and $y = 0$

Matches (b)

51. $y = Ca^x$

$(0, 2)$: $2 = Ca^0 = C$

$(3, 54)$: $54 = 2a^3$

$27 = a^3$

$3 = a$

$y = 2(3^x)$

52. $y = Ca^x$

$(1, 2): 2 = Ca$

$(2, 1): 1 = Ca^2$

Dividing eliminates C: $\frac{2}{1} = \frac{Ca}{Ca^2} = \frac{1}{a}$

So, $a = \frac{1}{2}$ and $C = 4$.

$y = 4\left(\frac{1}{2}\right)^x = 4\left(2^{-x}\right)$

53. $f(x) = \ln x + 1$

Vertical shift 1 unit upward
Matches (b)

54. $f(x) = -\ln x$

Reflection in the x-axis
Matches (d)

55. $f(x) = \ln(x - 1)$

Horizontal shift 1 unit to the right
Matches (a)

56. $f(x) = -\ln(-x)$

Reflection in the y-axis and the x-axis
Matches (c)

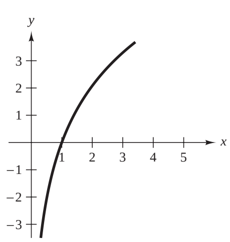
57. $e^0 = 1$
$\ln 1 = 0$

58. $e^{-2} = 0.1353\ldots$
$\ln 0.1353\ldots = -2$

59. $\ln 2 = 0.6931\ldots$
$e^{0.6931\ldots} = 2$

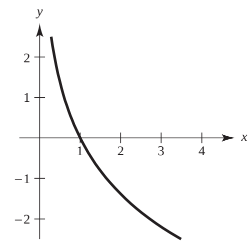
60. $\ln 0.5 = -0.6931\ldots$
$e^{-0.6931\ldots} = \frac{1}{2}$

61. $f(x) = 3 \ln x$



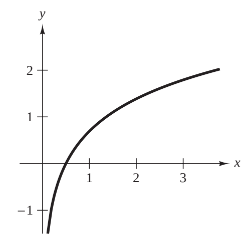
Domain: $x > 0$

62. $f(x) = -2 \ln x$



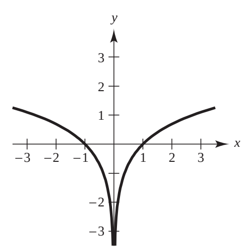
Domain: $x > 0$

63. $f(x) = \ln 2x$



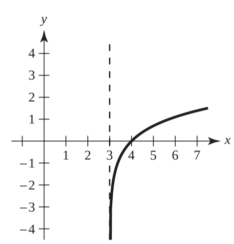
Domain: $x > 0$

64. $f(x) = \ln|x|$



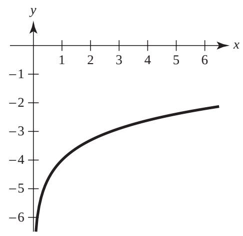
Domain: $x \neq 0$

65. $f(x) = \ln(x - 3)$



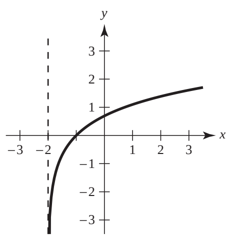
Domain: $x > 3$

66. $f(x) = \ln x - 4$



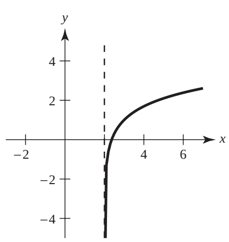
Domain: $x > 0$

67. $h(x) = \ln(x + 2)$



Domain: $x > -2$

68. $f(x) = \ln(x - 2) + 1$



Domain: $x > 2$

69. 8 units upward: $e^x + 8$

Reflected in x-axis: $-(e^x + 8)$

$y = -(e^x + 8) = -e^x - 8$

70. 2 units to the left: e^{x+2}

6 units downward: $e^{x+2} - 6$

$y = e^{x+2} - 6$

71. 5 units to the right: $\ln(x - 5)$

1 unit downward: $\ln(x - 5) - 1$

$y = \ln(x - 5) - 1$

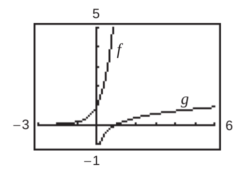
72. 3 units upward: $\ln x + 3$

Reflected in x-axis: $\ln(-x) + 3$

$y = \ln(-x) + 3$

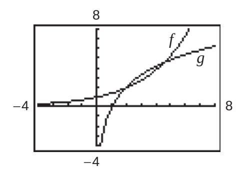
73. $f(x) = e^{2x}$

$g(x) = \ln\sqrt{x} = \dfrac{1}{2}\ln x$



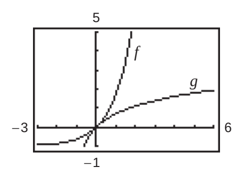
74. $f(x) = e^{x/3}$

$g(x) = \ln x^3 = 3\ln x$



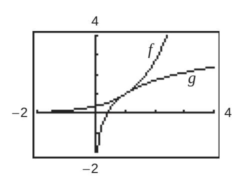
75. $f(x) = e^x - 1$

$g(x) = \ln(x + 1)$



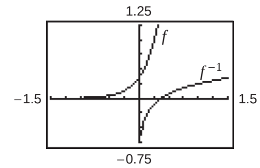
76. $f(x) = e^{x-1}$

$g(x) = 1 + \ln x$



77. (a)
$$\begin{aligned} y &= e^{4x-1} \\ \ln y &= 4x - 1 \\ \ln y + 1 &= 4x \\ x &= \tfrac{1}{4}(\ln y + 1) \\ f^{-1}(x) &= \tfrac{1}{4}(\ln x + 1) \end{aligned}$$

(b)

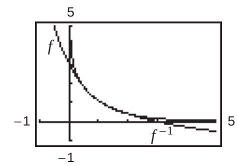


(c) $f^{-1}(f(x)) = f^{-1}(e^{4x-1}) = \tfrac{1}{4}(\ln e^{4x-1} + 1) = \tfrac{1}{4}(4x - 1 + 1) = x$

$f(f^{-1}(x)) = f\left(\tfrac{1}{4}(\ln x + 1)\right) = e^{(\ln x + 1) - 1} = e^{\ln x} = x$

78. (a)
$$\begin{aligned} y &= 3e^{-x} \\ \frac{y}{3} &= e^{-x} \\ \ln\frac{y}{3} &= -x \\ x &= -\ln\frac{y}{3} = \ln\frac{3}{y} \\ f^{-1}(x) &= \ln\frac{3}{x} = \ln 3 - \ln x \end{aligned}$$

(b)

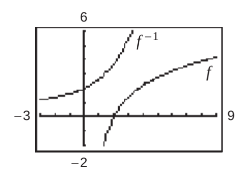


(c) $f^{-1}(f(x)) = f^{-1}(3e^{-x}) = \ln 3 - \ln(3e^{-x}) = \ln 3 - \ln 3 - \ln e^{-x} = x$

$f(f^{-1}(x)) = f\left(\ln\frac{3}{x}\right) = 3e^{-\ln(3/x)} = 3e^{\ln(3/x)} = 3\left(\frac{x}{3}\right) = x$

79. (a)
$$\begin{aligned} y &= 2\ln(x - 1) \\ \frac{y}{2} &= \ln(x - 1) \\ e^{y/2} &= x - 1 \\ x &= 1 + e^{y/2} \\ f^{-1}(x) &= 1 + e^{x/2} \end{aligned}$$

(b)

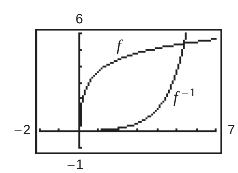


(c) $f^{-1}(f(x)) = f^{-1}(2\ln(x - 1)) = 1 + e^{\ln(x-1)} = 1 + x - 1 = x$

$f(f^{-1}(x)) = f(1 + x^{x/2}) = 2\ln\left[(1 + e^{x/2}) - 1\right] = 2\left(\frac{x}{2}\right) = x$

80. (a)
$$\begin{aligned} y &= 3 + \ln(2x) \\ y - 3 &= \ln 2x \\ e^{y-2} &= 2x \\ x &= \tfrac{1}{2}e^{y-3} \\ f^{-1}(x) &= \tfrac{1}{2}e^{x-3} \end{aligned}$$

(b)



(c) $f^{-1}(f(x)) = f^{-1}(3 + \ln(2x)) = \tfrac{1}{2}e^{3+\ln(2x)-3} = \tfrac{1}{2}(2x) = x$

$f(f^{-1}(x)) = f\left(\tfrac{1}{2}e^{x-3}\right) = 3 + \ln(e^{x-3}) = 3 + (x - 3) = x$

81. $\ln e^{x^2} = x^2$

82. $\ln e^{2x-1} = 2x - 1$

83. $e^{\ln(5x+2)} = 5x + 2$

84. $e^{\ln\sqrt{x}} = \sqrt{x}$

85. $-1 + \ln e^{2x} = -1 + 2x$

86. $-8 + e^{\ln x^3} = -8 + x^3$

87. (a) $\ln 6 = \ln 2 + \ln 3 \approx 1.7917$

(b) $\ln \frac{2}{3} = \ln 2 - \ln 3 \approx -0.4055$

(c) $\ln 81 = 4 \ln 3 \approx 4.3944$

(d) $\ln \sqrt{3} = \frac{1}{2} \ln 3 \approx 0.5493$

88. (a) $\ln 0.25 = \ln \frac{1}{4} = \ln 1 - 2 \ln 2 \approx -1.3862$

(b) $\ln 24 = 3 \ln 2 + \ln 3 \approx 3.1779$

(c) $\ln \sqrt[3]{12} = \frac{1}{3}(2 \ln 2 + \ln 3) \approx 0.8283$

(d) $\ln \frac{1}{72} = \ln 1 - (3 \ln 2 + 2 \ln 3) \approx -4.2765$

89. $\ln \dfrac{x}{4} = \ln x - \ln 4$

90. $\ln \sqrt{x^5} = \ln x^{5/2} = \frac{5}{2} \ln x$

91. $\ln \dfrac{xy}{z} = \ln x + \ln y - \ln z$

92. $\ln(xyz) = \ln x + \ln y + \ln z$

93. $\ln\left(x\sqrt{x^2 + 5}\right) = \ln x + \ln\left(x^2 + 5\right)^{1/2}$

$= \ln x + \frac{1}{2} \ln\left(x^2 + 5\right)$

94. $\ln \sqrt[3]{z + 1} = \ln(z + 1)^{1/3} = \frac{1}{3} \ln(z + 1)$

95. $\ln\sqrt{\dfrac{x - 1}{x}} = \ln\left(\dfrac{x - 1}{x}\right)^{1/2} = \dfrac{1}{2} \ln\left(\dfrac{x - 1}{x}\right)$

$= \dfrac{1}{2}\left[\ln(x - 1) - \ln x\right]$

$= \dfrac{1}{2} \ln(x - 1) - \dfrac{1}{2} \ln x$

96. $\ln z(z - 1)^2 = \ln z + \ln(z - 1)^2$

$= \ln z + 2 \ln(z - 1)$

97. $\ln 3e^2 = \ln 3 + 2 \ln e = 2 + \ln 3$

98. $\ln \dfrac{1}{e} = \ln 1 - \ln e = -1$

99. $\ln x + \ln 7 = \ln(x \cdot 7) = \ln(7x)$

100. $\ln y + \ln x^2 = \ln\left(yx^2\right)$

101. $\ln(x - 2) - \ln(x + 2) = \ln \dfrac{x - 2}{x + 2}$

102. $3 \ln x + 2 \ln y - 4 \ln z = \ln x^3 + \ln y^2 - \ln z^4$

$= \ln \dfrac{x^3 y^2}{z^4}$

103. $\dfrac{1}{3}\left[2 \ln(x + 3) + \ln x - \ln\left(x^2 - 1\right)\right] = \dfrac{1}{3} \ln \dfrac{x(x + 3)^2}{x^2 - 1}$

$= \ln \sqrt[3]{\dfrac{x(x + 3)^2}{x^2 - 1}}$

104. $2\left[\ln x - \ln(x + 1) - \ln(x - 1)\right] = 2 \ln \dfrac{x}{(x + 1)(x - 1)}$

$= \ln\left(\dfrac{x}{x^2 - 1}\right)^2$

105. $2 \ln 3 - \dfrac{1}{2} \ln\left(x^2 + 1\right) = \ln 9 - \ln\sqrt{x^2 + 1} = \ln \dfrac{9}{\sqrt{x^2 + 1}}$

106. $\dfrac{3}{2}\left[\ln\left(x^2 + 1\right) - \ln(x + 1) - \ln(x - 1)\right] = \dfrac{3}{2} \ln \dfrac{x^2 + 1}{(x + 1)(x - 1)}$

$= \ln \sqrt{\left(\dfrac{x^2 + 1}{x^2 - 1}\right)^3}$

107. (a) $e^{\ln x} = 4$

$x = 4$

(b) $\ln e^{2x} = 3$

$2x = 3$

$x = \frac{3}{2}$

108. (a) $e^{\ln 2x} = 12$

$2x = 12$

$x = 6$

(b) $\ln e^{-x} = 0$

$-x = 0$

$x = 0$

109. (a) $\ln x = 2$

$x = e^2 \approx 7.389$

(b) $e^x = 4$

$x = \ln 4 \approx 1.386$

110. (a) $\ln x^2 = 8$

$x^2 = e^8$

$x = \pm e^4 \approx \pm 54.598$

(b) $e^{-2x} = 5$

$-2x = \ln 5$

$x = -\frac{1}{2} \ln 5 \approx -0.805$

111. $e^x > 5$

$\ln e^x > \ln 5$

$x > \ln 5$

112. $e^{1-x} < 6$

$\ln e^{1-x} < \ln 6$

$1 - x < \ln 6$

$x > 1 - \ln 6$

113. $-2 < \ln x < 0$

$e^{-2} < x < e^0 = 1$

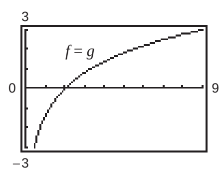
$\frac{1}{e^2} < x < 1$

114. $1 < \ln x < 100$

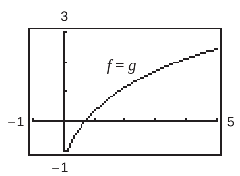
$e^1 < x < e^{100}$

$e < x < e^{100}$

115.



116.



117. The domain of the natural logarithmic function is $(0, \infty)$ and the range is $(-\infty, \infty)$. The function is continuous, increasing, and one-to-one, and its graph is concave downward. In addition, if a and b are positive numbers and n is rational, then $\ln(1) = 0$, $\ln(a \cdot b) = \ln a + \ln b$, $\ln(a^n) = n \ln a$, and $\ln(a/b) = \ln a - \ln b$.

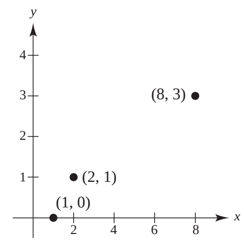
118. The functions $f(x) = e^x$ and $g(x) = \ln x$ are inverses of each other. So, $\ln e^x = g(f(x)) = x$.

119. $f(x) = e^x$. Domain is $(-\infty, \infty)$ and range is $(0, \infty)$. f is continuous, increasing, one-to-one, and concave upwards on its entire domain.

$\lim_{x\to-\infty} e^x = 0$ and $\lim_{x\to\infty} e^x = \infty$

120. The graphs of $f(x) = \ln x$ and $g(x) = e^x$ are mirror images in the line $y = x$.

121.



x	1	2	8
y	0	1	3

(a) y is an exponential function of x: False

(b) y is a logarithmic function of x: True; $y = \log_2 x$

(c) x is an exponential function of y: True; $2^y = x$

(d) y is a linear function of x: False

122. The graph is that of $y_2 = e^{\ln x}$.

The domain of $y_1 = \ln(e^x)$ is $(-\infty, \infty)$.

The domain of $y_2 = e^{\ln x}$ is $x > 0$.

No, $\ln e^x \neq e^{\ln x}$ for all real values of x. They are equal for $x > 0$.

123. (a) $\beta = \frac{10}{\ln 10} \ln\left(\frac{I}{10^{-16}}\right)$

$= \frac{10}{\ln 10}\left[\ln I - \ln 10^{-16}\right]$

$= \frac{10}{\ln 10}\left[\ln I + 16 \ln 10\right]$

$= \frac{10}{\ln 10} \ln I + 160$

$= 10 \log_{10} I + 160$

124. $\beta(10^{-5}) = \frac{10}{\ln 10} \ln 10^{-5} + 160$

$= -50 + 160 = 110$ decibels

125. False

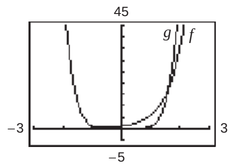
$\ln x + \ln 25 = \ln(25x) \neq \ln(x + 25)$

126. False. The property is

$\ln xy = \ln x + \ln y$ (for $x, y > 0$). As a counter example, let $x = y = e$. Then

$\ln xy = \ln e^2 = 2$ and $\ln x \ln y = 1 \cdot 1 = 1$.

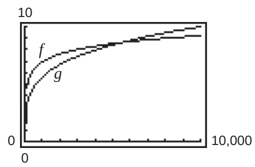
127.



The graphs intersect three times: $(-0.7899,\ 0.2429)$, $(1.6242,\ 18.3615)$ and $(6,\ 46{,}656)$.

The function $f(x) = 6^x$ grows more rapidly.

128.

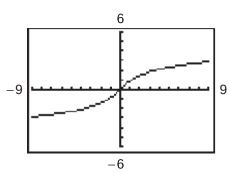


The graphs intersect twice: $(4.1771,\ 1.4296)$ and $(5503.647,\ 8.6132)$.

$g(x) = x^{1/4}$ grows more rapidly.

129. $f(x) = \ln\left(x + \sqrt{x^2 + 1}\right)$

(a)



Domain: $-\infty < x < \infty$

(b)
$$\begin{aligned} f(-x) &= \ln\left(-x + \sqrt{x^2+1}\right) \\ &= \ln\left[\frac{\left(-x + \sqrt{x^2+1}\right)\left(-x - \sqrt{x^2+1}\right)}{\left(-x - \sqrt{x^2+1}\right)}\right] \\ &= \ln\left[\frac{\left(x^2\sqrt{x^2+1}\right)}{\left(-x - \sqrt{x^2+1}\right)}\right] \\ &= \ln\left[\frac{1}{\left(x + \sqrt{x^2+1}\right)}\right] \\ &= -\ln\left(x + \sqrt{x^2+1}\right) = -f(x) \end{aligned}$$

(c)
$$\begin{aligned} y &= \ln\left(x + \sqrt{x^2+1}\right) \\ e^y &= x + \sqrt{x^2+1} \\ \left(e^y - x\right)^2 &= x^2 + 1 \\ 2xe^y &= e^{2y} - 1 \\ x &= \frac{e^{2y} - 1}{2e^x} \end{aligned}$$

130. $p(x) = \dfrac{x}{\ln x}$

$$p'(x) = \frac{(\ln x)(1) - x\left(\frac{1}{x}\right)}{(\ln x)^2} = \frac{\ln x - 1}{(\ln x)^2}$$

(a) $p'(1000) = \dfrac{\ln 1000 - 1}{(\ln 1000)^2} \approx 0.1238$

About 12.4 primes per 100 integers

(b) $p'(1{,}000{,}000) = \dfrac{\ln(1{,}000{,}000) - 1}{(\ln 1{,}000{,}000)^2} \approx 0.0671$

About 6.7 primes per 100 integers

(c) $p'(1{,}000{,}000{,}000) = \dfrac{\ln(1{,}000{,}000{,}000) - 1}{(\ln 1{,}000{,}000{,}000)^2}$
≈ 0.0459

About 4.6 primes per 100 integers

131. $n = 12$

$12! = 12 \cdot 11 \cdot 10 \cdots 3 \cdot 2 \cdot 1 = 479{,}001{,}600$

Stirlings Formula:

$12! \approx \left(\dfrac{12}{e}\right)^{12}\sqrt{2\pi(12)} \approx 475{,}687{,}487$

132. $n = 15$

$15! = 15 \cdot 14 \cdots 3 \cdot 2 \cdot 1 = 1{,}307{,}674{,}368{,}000$

Stirlings Formula:

$15! \approx \left(\dfrac{15}{e}\right)^{15}\sqrt{2\pi(15)} \approx 1{,}300{,}430{,}722{,}200$
$\approx 1.3004 \times 10^{12}$

Review Exercises for Chapter 1

1. $y = 5x - 8$

$x = 0$: $y = 5(0) - 8 = -8 \Rightarrow (0, -8)$, y-intercept

$y = 0$: $0 = 5x - 8 \Rightarrow x = \frac{8}{5} \Rightarrow \left(\frac{8}{5}, 0\right)$, x-intercept

2. $y = x^2 - 8x + 12$

$x = 0$: $y = (0)^2 - 8(0) + 12 = 12 \Rightarrow (0, 12)$, y-intercept

$y = 0$: $x^2 - 8x + 12 = (x - 6)(x - 2) = 0 \Rightarrow x = 2, 6 \Rightarrow (2, 0), (6, 0)$, x-intercepts

3. $y = \dfrac{x - 3}{x - 4}$

$x = 0$: $y = \dfrac{0 - 3}{0 - 4} = \dfrac{3}{4} \Rightarrow \left(0, \dfrac{3}{4}\right)$, y-intercept

$y = 0$: $0 = \dfrac{x - 3}{x - 4} \Rightarrow x = 3 \Rightarrow (3, 0)$, x-intercept

4. $y = (x - 3)\sqrt{x + 4}$

$x = 0$: $y = (0 - 3)\sqrt{0 + 4} = -3\sqrt{4} = -3(2) = -6 \Rightarrow (0, -6)$, y-intercept

$y = 0$: $(x - 3)\sqrt{x + 4} = 0 \Rightarrow x = 3, -4 \Rightarrow (3, 0), (-4, 0)$, x-intercepts

5. $y = x^2 + 4x$ does not have symmetry with respect to either axis or the origin.

6. Symmetric with respect to y-axis because

$y = (-x)^4 - (-x)^2 + 3$

$y = x^4 - x^2 + 3.$

7. Symmetric with respect to both axes and the origin because:

$y^2 = (-x^2) - 5$	$(-y)^2 = x^2 - 5$	$(-y)^2 = (-x)^2 - 5$
$y^2 = x^2 - 5$	$y^2 = x^2 - 5$	$y^2 = x^2 - 5$

8. Symmetric with respect to the origin because:

$(-x)(-y) = -2$

$xy = -2.$

9. $y = -\dfrac{1}{2}x + 3$

y-intercept: $y = -\dfrac{1}{2}(0) + 3 = 3$

$(0, 3)$

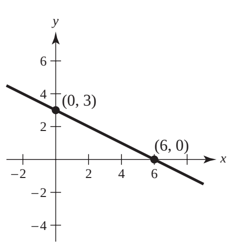
x-intercept: $-\dfrac{1}{2}x + 3 = 0$

$-\dfrac{1}{2}x = -3$

$x = 6$

$(6, 0)$

Symmetry: none



10. $y = -x^2 + 4$

y-intercept: $y = -(0)^2 + 4 = 4$

$(0, 4)$

x-intercepts: $-x^2 + 4 = 0$

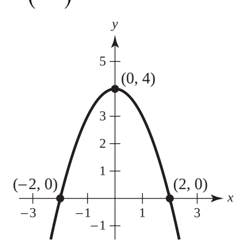
$(2 - x)(2 + x) = 0$

$x = \pm 2$

$(2, 0), (-2, 0)$

Symmetric with respect to the y-axis because

$-(-x)^2 + 4 = -x^2 + 4.$



11. $y = x^3 - 4x$

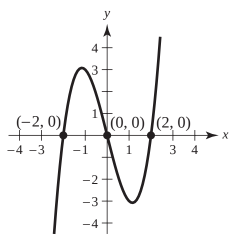
y-intercept: $y = 0^3 - 4(0) = 0$

$(0, 0)$

x-intercepts:
$$\begin{aligned} x^3 - 4x &= 0 \\ x(x^2 - 4) &= 0 \\ x(x - 2)(x + 2) &= 0 \\ x &= 0, 2, -2 \end{aligned}$$

$(0, 0), (2, 0), (-2, 0)$

Symmetric with respect to the origin because $(-x)^3 - 4(-x) = -x^3 + 4x = -(x^3 - 4x)$.



12. $y^2 = 9 - x$

$y^2 + x - 9 = 0$

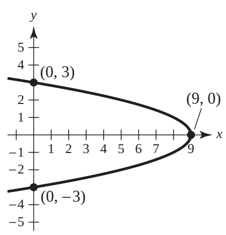
y-intercept: $y^2 = 9 - 0 = 9 \Rightarrow y = \pm 3$

$(0, 3), (0, -3)$

x-intercept: $0^2 = 9 - x \Rightarrow x = 9$

$(9, 0)$

Symmetric with respect to the x-axis because $(-y)^2 + x - 9 = y^2 + x - 9 = 0$.



13. $y = 2\sqrt{4 - x}$

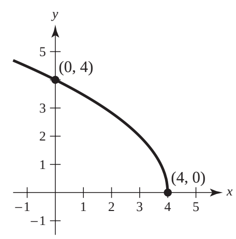
y-intercept: $y = 2\sqrt{4 - 0} = 2\sqrt{4} = 4$

$(0, 4)$

x-intercept:
$$\begin{aligned} 2\sqrt{4 - x} &= 0 \\ \sqrt{4 - x} &= 0 \\ 4 - x &= 0 \\ x &= 4 \end{aligned}$$

$(4, 0)$

Symmetry: none



14. $y = |x - 4| - 4$

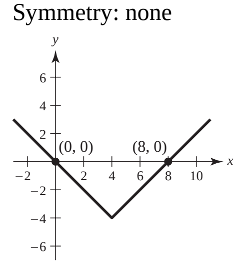
y-intercept: $y = |0 - 4| - 4 = |-4| - 4 = 4 - 4 = 0$

$(0, 0)$

x-intercepts:
$$\begin{aligned} |x - 4| - 4 &= 0 \\ |x - 4| &= 4 \\ x - 4 = 4 \quad &\text{or} \quad x - 4 = -4 \\ x = 8 \quad &\quad\; x = 0 \end{aligned}$$

$(0, 0), (8, 0)$

Symmetry: none



15. $5x + 3y = -1 \Rightarrow y = \frac{1}{3}(-5x - 1)$

$x - y = -5 \Rightarrow y = x + 5$

$$\begin{aligned} \tfrac{1}{3}(-5x - 1) &= x + 5 \\ -5x - 1 &= 3x + 15 \\ -16 &= 8x \\ -2 &= x \end{aligned}$$

For $x = -2$, $y = x + 5 = -2 + 5 = 3$.

Point of intersection is: $(-2, 3)$

16. $2x + 4y = 9 \Rightarrow y = \dfrac{-2x + 9}{4}$

$6x - 4y = 7 \Rightarrow y = \dfrac{6x - 7}{4}$

$\dfrac{-2x + 9}{4} = \dfrac{6x - 7}{4}$

$-2x + 9 = 6x - 7$

$-8x = -16$

$x = 2$

For $x = 2$, $y = \dfrac{6(2) - 7}{4} = \dfrac{5}{4}$

Point of intersection: $\left(2, \dfrac{5}{4}\right)$

17. $x - y = -5 \Rightarrow y = x + 5$

$x^2 - y = 1 \Rightarrow y = x^2 - 1$

$x + 5 = x^2 - 1$

$0 = x^2 - x - 6$

$0 = (x - 3)(x + 2)$

$x = 3$ or $x = -2$

For $x = 3$, $y = 3 + 5 = 8$.

For $x = -2$, $y = -2 + 5 = 3$.

Points of intersection: $(3, 8)$, $(-2, 3)$

18. $x^2 + y^2 = 1 \Rightarrow y^2 = 1 - x^2$

$-x + y = 1 \Rightarrow y = x + 1$

$1 - x^2 = (x + 1)^2$

$1 - x^2 = x^2 + 2x + 1$

$0 = 2x^2 + 2x$

$0 = 2x(x + 1)$

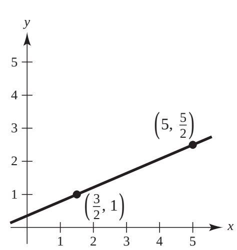
$x = 0$ or $x = -1$

For $x = 0$, $y = 0 + 1 = 1$.

For $x = -1$, $y = -1 + 1 = 0$.

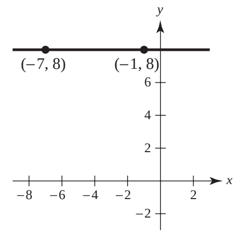
Points of intersection: $(0, 1)$, $(-1, 0)$

19.



Slope $= \dfrac{\left(\frac{5}{2}\right) - 1}{5 - \left(\frac{3}{2}\right)} = \dfrac{\frac{3}{2}}{\frac{7}{2}} = \dfrac{3}{7}$

20. The line is horizontal and has slope 0.

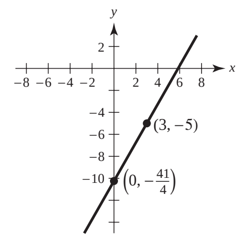


21. $y - (-5) = \frac{7}{4}(x - 3)$

$y + 5 = \frac{7}{4}x - \frac{21}{4}$

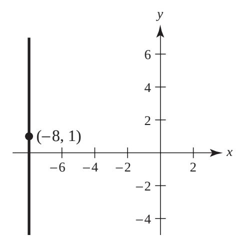
$4y + 20 = 7x - 21$

$0 = 7x - 4y - 41$



22. Because m is undefined the line is vertical.

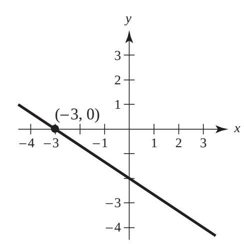
$x = -8$ or $x + 8 = 0$



23. $y - 0 = -\frac{2}{3}(x - (-3))$

$y = -\frac{2}{3}x - 2$

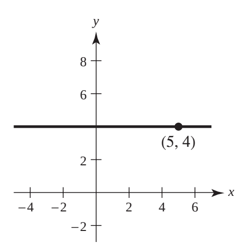
$2x + 3y + 6 = 0$



24. Because $m = 0$, the line is horizontal.

$y - 4 = 0(x - 5)$

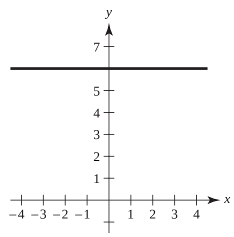
$y = 4$ or $y - 4 = 0$



25. $y = 6$

Slope: 0

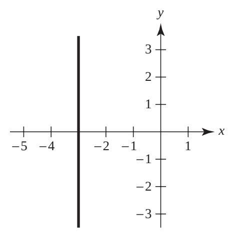
y-intercept: $(0, 6)$



26. $x = -3$

Slope: undefined

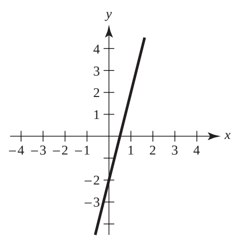
Line is vertical.



27. $y = 4x - 2$

Slope: 4

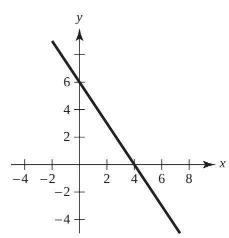
y-intercept: $(0, -2)$



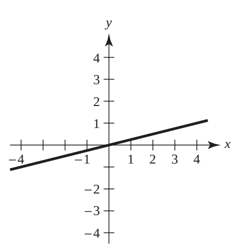
28.
$$\begin{aligned} 3x + 2y &= 12 \\ 2y &= -3x + 12 \\ y &= \frac{-3}{2}x + 6 \end{aligned}$$

Slope : $-\dfrac{3}{2}$

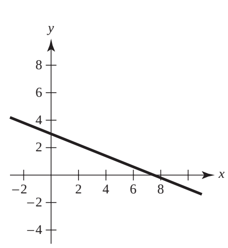
y-intercept: $(0, 6)$



29.
$$\begin{aligned} m &= \frac{2 - 0}{8 - 0} = \frac{1}{4} \\ y - 0 &= \frac{1}{4}(x - 0) \\ y &= \frac{1}{4}x \\ 4y - x &= 0 \end{aligned}$$



30.
$$\begin{aligned} m &= \frac{-1 - 5}{10 - (-5)} = \frac{-6}{15} = -\frac{2}{5} \\ y - 5 &= \frac{-2}{5}(x - (-5)) \\ 5y - 25 &= -2x - 10 \\ 5y + 2x - 15 &= 0 \end{aligned}$$



31. (a)
$$\begin{aligned} y - 5 &= \tfrac{7}{16}(x + 3) \\ 16y - 80 &= 7x + 21 \\ 0 &= 7x - 16y + 101 \end{aligned}$$

(b) $5x - 3y = 3$ has slope $\frac{5}{3}$.

$$\begin{aligned} y - 5 &= \tfrac{5}{3}(x + 3) \\ 3y - 15 &= 5x + 15 \\ 0 &= 5x - 3y + 30 \end{aligned}$$

(c)
$$\begin{aligned} 3x + 4y &= 8 \\ 4y &= -3x + 8 \\ y &= \frac{-3}{4}x + 2 \end{aligned}$$

Perpendicular line has slope $\dfrac{4}{3}$.

$$\begin{aligned} y - 5 &= \frac{4}{3}(x - (-3)) \\ 3y - 15 &= 4x + 12 \end{aligned}$$

$4x - 3y + 27 = 0$ or $y = \dfrac{4}{3}x + 9$

(d) Slope is undefined so the line is vertical.

$$\begin{aligned} x &= -3 \\ x + 3 &= 0 \end{aligned}$$

32. (a)
$$\begin{aligned} y - 4 &= -\frac{2}{3}(x - 2) \\ 3y - 12 &= -2x + 4 \\ 2x + 3y - 16 &= 0 \end{aligned}$$

(b) $x + y = 0$ has slope -1. Slope of the perpendicular line is 1.

$$\begin{aligned} y - 4 &= 1(x - 2) \\ y &= x + 2 \\ 0 &= x - y + 2 \end{aligned}$$

(c) $m = \dfrac{4 - 1}{2 - 6} = -\dfrac{3}{4}$

$$\begin{aligned} y - 4 &= -\frac{3}{4}(x - 2) \\ 4y - 16 &= -3x + 6 \\ 3x + 4y - 22 &= 0 \end{aligned}$$

(d) Because the line is horizontal the slope is 0.

$$\begin{aligned} y &= 4 \\ y - 4 &= 0 \end{aligned}$$

33. The slope is -850.

$V = -850t + 12{,}500.$

$V(3) = -850(3) + 12{,}500 = \$9950$

34. (a) $C = 9.25t + 13.50t + 36{,}500 = 22.75t + 36{,}500$

(b) $R = 30t$

(c) $$\begin{aligned} 30t &= 22.75t + 36{,}500 \\ 7.25t &= 36{,}500 \\ t &\approx 5034.48 \text{ hours to break even} \end{aligned}$$

35. $f(x) = 5x + 4$

(a) $f(0) = 5(0) + 4 = 4$

(b) $f(5) = 5(5) + 4 = 29$

(c) $f(-3) = 5(-3) + 4 = -11$

(d) $f(t + 1) = 5(t + 1) + 4 = 5t + 9$

36. $f(x) = x^3 - 2x$

(a) $f(-3) = (-3)^3 - 2(-3) = -27 + 6 = -21$

(b) $f(2) = 2^3 - 2(2) = 8 - 4 = 4$

(c) $f(-1) = (-1)^3 - 2(-1) = -1 + 2 = 1$

(d) $$\begin{aligned} f(c - 1) &= (c - 1)^3 - 2(c - 1) \\ &= c^3 - 3c^2 + 3c - 1 - 2c + 2 \\ &= c^3 - 3c^2 + c + 1 \end{aligned}$$

37. $f(x) = 4x^2$

$$\begin{aligned} \frac{f(x + \Delta x) - f(x)}{\Delta x} &= \frac{4(x + \Delta x)^2 - 4x^2}{\Delta x} \\ &= \frac{4\left(x^2 + 2x\Delta x + (\Delta x)^2\right) - 4x^2}{\Delta x} \\ &= \frac{4x^2 + 8x\Delta x + 4(\Delta x)^2 - 4x^2}{\Delta x} \\ &= \frac{8x\Delta x + 4(\Delta x)^2}{\Delta x} \\ &= 8x + 4\Delta x, \quad \Delta x \neq 0 \end{aligned}$$

38. $f(x) = 2x - 6$

$f(1) = 2(1) - 6 = -4$

$$\begin{aligned} \frac{f(x) - f(-1)}{x - 1} &= \frac{(2x - 6) - (-4)}{x - 1} \\ &= \frac{2x - 6 + 4}{x - 1} \\ &= \frac{2x - 2}{x - 1} \\ &= \frac{2(x - 1)}{x - 1} \\ &= 2, \quad x \neq 1 \end{aligned}$$

39. $f(x) = x^2 + 3$

Domain: $(-\infty, \infty)$

Range: $[3, \infty)$

40. $g(x) = \sqrt{6 - x}$

Domain: $6 - x \geq 0$

$6 \geq x$

$(-\infty, 6]$

Range: $[0, \infty)$

41. $f(x) = -|x + 1|$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 0]$

42. $h(x) = \dfrac{2}{x + 1}$

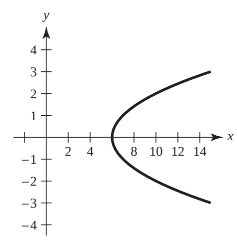
Domain: all $x \neq -1$; $(-\infty, -1) \cup (-1, \infty)$

Range: all $y \neq 0$; $(-\infty, 0) \cup (0, \infty)$

43. $x - y^2 = 6$

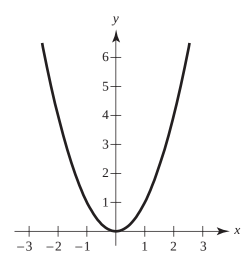
$y = \pm\sqrt{x - 6}$

Not a function because there are two values of y for some x.



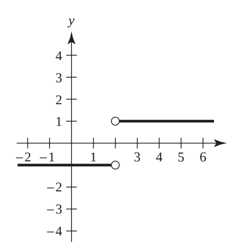
44. $x^2 - y = 0$

Function of x because there is one value for y for each x.



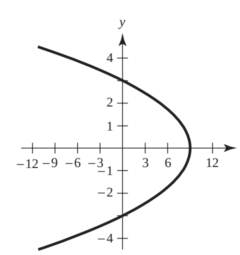
45. $y = \dfrac{|x - 2|}{x - 2}$

y is a function of x because there is one value of y for each x.

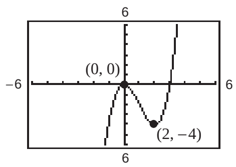


46. $x = 9 - y^2$

Not a function of x since there are two values of y for some x.



47. $f(x) = x^3 - 3x^2$



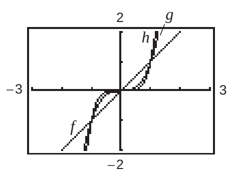
(a) The graph of g is obtained from f by a vertical shift down 1 unit, followed by a reflection in the x-axis:

$$g(x) = -[f(x) - 1] = -x^3 + 3x^2 + 1$$

(b) The graph of g is obtained from f by a vertical shift upwards of 1 and a horizontal shift of 2 to the right.

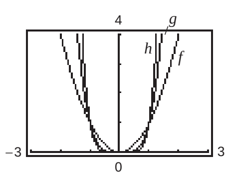
$$g(x) = f(x - 2) + 1 = (x - 2)^3 - 3(x - 2)^2 + 1$$

48. (a) Odd powers: $f(x) = x,\ g(x) = x^3,\ h(x) = x^5$



The graphs of f, g, and h all rise to the right and fall to the left. As the degree increases, the graph rises and falls more steeply. All three graphs pass through the points (0, 0), (1, 1), and $(-1, -1)$ and are symmetric with respect to the origin.

Even powers: $f(x) = x^2,\ g(x) = x^4,\ h(x) = x^6$



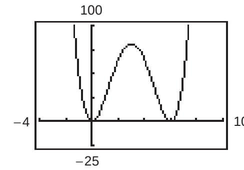
The graphs of f, g, and h all rise to the left and to the right. As the degree increases, the graph rises more steeply. All three graphs pass through the points $(0, 0)$, $(1, 1)$, and $(-1, 1)$ and are symmetric with respect to the y-axis.

All of the graphs, even and odd, pass through the origin. As the powers increase, the graphs become flatter in the interval $-1 < x < 1$.

(b) $y = x^7$ will look like $h(x) = x^5$, but rise and fall even more steeply. $y = x^8$ will look like $h(x) = x^6$, but rise even more steeply.

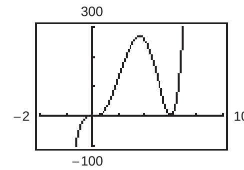
49. (a) $f(x) = x^2(x - 6)^2$

The leading coefficient is positive and the degree is even so the graph will rise to the left and to the right.



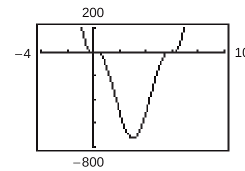
(b) $g(x) = x^3(x - 6)^2$

The leading coefficient is positive and the degree is odd so the graph will rise to the right and fall to the left.



(c) $h(x) = x^3(x - 6)^3$

The leading coefficient is positive and the degree is even so the graph will rise to the left and to the right.



50. (a) 3 (cubic), negative leading coefficient

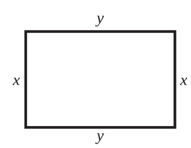
(b) 4 (quartic), positive leading coefficient

(c) 2 (quadratic), negative leading coefficient

(d) 5, positive leading coefficient

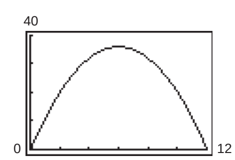
51. For company (a) the profit rose rapidly for the first year, and then leveled off. For the second company (b), the profit dropped, and then rose again later.

52. (a)



$$2x + 2y = 24$$
$$y = 12 - x$$
$$A = xy = x(12 - x)$$

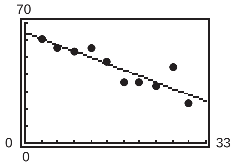
(b) Domain: $0 < x < 12$ or $(0, 12)$



(c) Maximum area is $A = 36$ in.^{2}. In general, the maximum area is attained when the rectangle is a square. In this case, $x = 6$.

53. (a) $y = -1.204x + 64.2667$

(b)

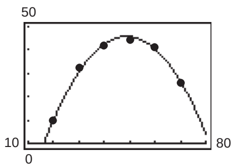


(c) The data point (27, 44) is probably an error. Without this point, the new model is $y = -1.4344x + 66.4387$.

54. (a) Using a graphing utility, you obtain

$y = -0.043x^2 + 4.19x - 56.2.$

(b)



(c) For $x = 26$:

$$y = -0.043(26)^2 + 4.19(26) - 56.2$$
$$\approx \$23.7 \text{ thousand}$$

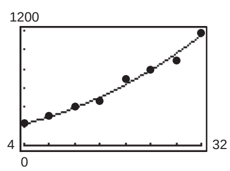
(d) For $x = 34$:

$$y = -0.043(34)^2 + 4.19(34) - 56.2$$
$$\approx \$36.6 \text{ thousand}$$

55. (a) Using a graphing utility,

$y = 0.61t^2 + 11.0t + 172$

(b)



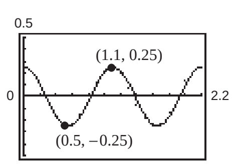
The model fits the data well.

56. (a) Yes, y is a function of t. At each time t, there is one and only one displacement y.

(b) The amplitude is approximately $(0.25 - (-0.25))/2 = 0.25$. The period is approximately 1.1.

(c) One model is $y = \frac{1}{4}\cos\left(\frac{2\pi}{1.1}t\right) \approx \frac{1}{4}\cos(5.7t)$

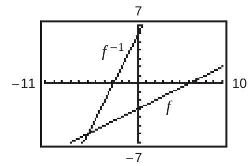
(d)



The model appears to fit the data.

57. (a)
$$f(x) = \tfrac{1}{2}x - 3$$
$$y = \tfrac{1}{2}x - 3$$
$$2(y + 3) = x$$
$$2(x + 3) = y$$
$$f^{-1}(x) = 2x + 6$$

(b)

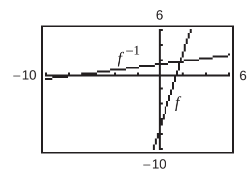


(c) $f^{-1}(f(x)) = f^{-1}\left(\tfrac{1}{2}x - 3\right) = 2\left(\tfrac{1}{2}x - 3\right) + 6 = x$

$f(f^{-1}(x)) = f(2x + 6) = \tfrac{1}{2}(2x + 6) - 3 = x$

58. (a)
$$f(x) = 5x - 7$$
$$y = 5x - 7$$
$$\frac{y + 7}{5} = x$$
$$\frac{x + 7}{5} = y$$
$$f^{-1}(x) = \frac{x + 7}{5}$$

(b)

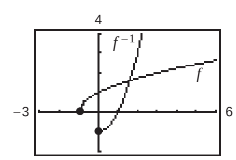


(c) $f^{-1}(f(x)) = f^{-1}(5x - 7) = \dfrac{(5x - 7) + 7}{5} = x$

$f(f^{-1}(x)) = f\left(\dfrac{x + 7}{5}\right) = 5\left(\dfrac{x + 7}{5}\right) - 7 = x$

59. (a)
$$f(x) = \sqrt{x + 1}$$
$$y = \sqrt{x + 1}$$
$$y^2 - 1 = x$$
$$x^2 - 1 = y$$
$$f^{-1}(x) = x^2 - 1, \quad x \ge 0$$

(b)



(c) $f^{-1}(f(x)) = f^{-1}\left(\sqrt{x + 1}\right) = \sqrt{(x^2 - 1)^2} - 1 = x$

$f(f^{-1}(x)) = f(x^2 - 1) = \sqrt{(x^2 - 1) + 1}$

$= \sqrt{x^2} = x \text{ for } x \ge 0$

60. (a) $f(x) = x^3 + 2$

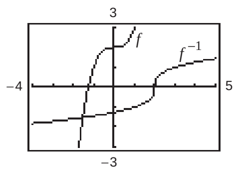
$y = x^3 + 2$

$\sqrt[3]{y - 2} = x$

$\sqrt[3]{x - 2} = y$

$f^{-1}(x) = \sqrt[3]{x - 2}$

(b)



(c) $f^{-1}(f(x)) = f^{-1}(x^3 + 2) = \sqrt[3]{(x^3 + 2) - 2} = x$

$f(f^{-1}(x)) = f(\sqrt[3]{x - 2}) = (\sqrt[3]{x - 2})^3 + 2 = x$

61. (a) $f(x) = \sqrt[3]{x + 1}$

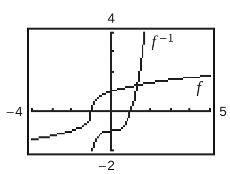
$y = \sqrt[3]{x + 1}$

$y^3 - 1 = x$

$x^3 - 1 = y$

$f^{-1}(x) = x^3 - 1$

(b)



(c) $f^{-1}(f(x)) = f^{-1}(\sqrt[3]{x + 1})$

$= (\sqrt[3]{x + 1})^3 - 1 = x$

$f(f^{-1}(x)) = f(x^3 - 1) = \sqrt[3]{(x^3 - 1) + 1} = x$

62. (a) $f(x) = x^2 - 5, \quad x \ge 0$

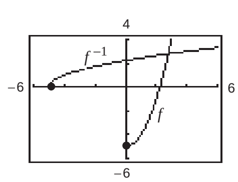
$y = x^2 - 5$

$\sqrt{y + 5} = x$

$\sqrt{x + 5} = y$

$f^{-1}(x) = \sqrt{x + 5}$

(b)

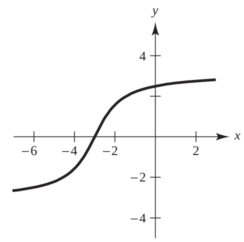


(c) $f^{-1}(f(x)) = f^{-1}(x^2 - 5)$

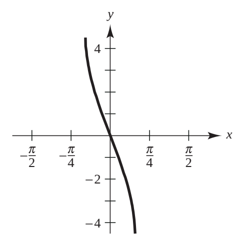
$= \sqrt{(x^2 - 5) + 5} = x$ for $x \ge 0$.

$f(f^{-1}(x)) = f(\sqrt{x + 5}) = (\sqrt{x + 5})^2 - 5 = x$

63. $f(x) = 2\arctan(x + 3)$



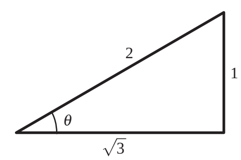
64. $h(x) = -3\arcsin(2x)$



65. Let $\theta = \arcsin \frac{1}{2}$.

$\sin \theta = \frac{1}{2}$

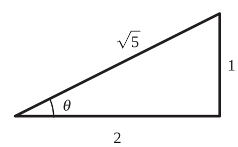
$\sin\left(\arcsin \frac{1}{2}\right) = \sin \theta = \frac{1}{2}$



66. Let $\theta = \operatorname{arccot} 2$.

$\cot \theta = 2$

$\tan(\operatorname{arccot}) = \tan \theta = \frac{1}{2}$



67. $f(x) = e^x$ matches (d).

The graph is increasing and the domain is all real x.

68. $f(x) = e^{-x}$ matches (a).

The graph is decreasing and the domain is all real x.

69. $f(x) = \ln(x + 1) + 1$ matches (c).

The graph is increasing and the domain is $x > -1$.

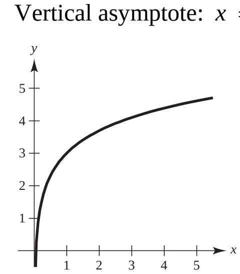
70. $f(x) = -\ln(x + 1) + 1$ matches (b).

The graph is decreasing and the domain is $x > -1$.

71. $f(x) = \ln x + 3$

Vertical shift three units upward

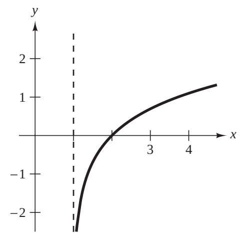
Vertical asymptote: $x = 0$



72. $f(x) = \ln(x - 1)$

Horizontal shift one unit to the right

Vertical asymptote: $x = 1$



73. $\ln \sqrt[5]{\dfrac{4x^2 - 1}{4x^2 + 1}} = \dfrac{1}{5} \ln \dfrac{(2x - 1)(2x + 1)}{4x^2 + 1}$

$= \dfrac{1}{5}\left[\ln(2x - 1) + \ln(2x + 1) - \ln(4x^2 + 1)\right]$

74. $\ln\left[(x^2 + 1)(x - 1)\right] = \ln(x^2 + 1) + \ln(x - 1)$

75. $\ln 3 + \dfrac{1}{3}\ln(4 - x^2) - \ln x = \ln 3 + \ln \sqrt[3]{4 - x^2} - \ln x$

$= \ln\left(\dfrac{3\sqrt[3]{4 - x^2}}{x}\right)$

76. $3\left[\ln x - 2\ln(x^2 + 1)\right] + 2\ln 5 = 3\ln x - 6\ln(x^2 + 1) + \ln 5^2$

$= \ln x^3 - \ln(x^2 + 1)^6 + \ln 25 = \ln\left[\dfrac{25x^3}{(x^2 + 1)^6}\right]$

77. $\ln\sqrt{x + 1} = 2$

$\sqrt{x + 1} = e^2$

$x + 1 = e^4$

$x = e^4 - 1 \approx 53.598$

78. $\ln x + \ln(x - 3) = 0$

$\ln x(x - 3) = 0$

$x(x - 3) = e^0$

$x^2 - 3x - 1 = 0$

$x = \dfrac{3 \pm \sqrt{13}}{2}$

$x = \dfrac{3 + \sqrt{13}}{2}$ only because $\dfrac{3 - \sqrt{13}}{2} < 0.$

79. (a) $f(x) = \ln\sqrt{x}$

$y = \ln\sqrt{x}$

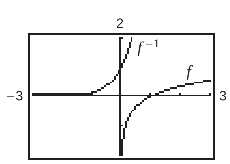
$e^y = \sqrt{x}$

$e^{2y} = x$

$e^{2x} = y$

$f^{-1}(x) = e^{2x}$

(b)



(c) $f^{-1}(f(x)) = f^{-1}(\ln\sqrt{x}) = e^{2\ln\sqrt{x}} = e^{\ln x} = x$

$f(f^{-1}(x)) = f(e^{2x}) = \ln\sqrt{e^{2x}} = \ln e^x = x$

80. (a) $f(x) = e^{1-x}$

$y = e^{1-x}$

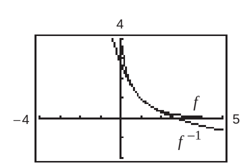
$\ln y = 1 - x$

$x = 1 - \ln y$

$y = 1 - \ln x$

$f^{-1}(x) = 1 - \ln x$

(b)

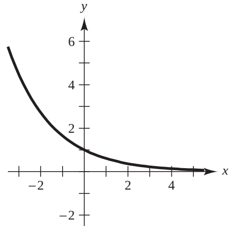


(c) $f^{-1}(f(x)) = f^{-1}(e^{1-x}) = 1 - \ln(e^{1-x})$

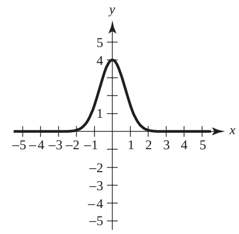
$= 1 - (1 - x) = x$

$f(f^{-1}(x)) = f(1 - \ln x) = e^{1-(1-\ln x)} = e^{\ln x} = x$

81. $f = e^{-x/2}$



82. $f = 4e^{-x^2}$



Problem Solving for Chapter 1

1. (a)
$$x^2 - 6x + y^2 - 8y = 0$$
$$(x^2 - 6x + 9) + (y^2 - 8y + 16) = 9 + 16$$
$$(x - 3)^2 + (y - 4)^2 = 25$$

Center: (3, 4); Radius: 5

(b) Slope of line from (0, 0) to (3, 4) is $\frac{4}{3}$. Slope of tangent line is $-\frac{3}{4}$. So, $y - 0 = -\frac{3}{4}(x - 0) \Rightarrow y = -\frac{3}{4}x$ Tangent line

(c) Slope of line from (6, 0) to (3, 4) is $\frac{4 - 0}{3 - 6} = -\frac{4}{3}$.

Slope of tangent line is $\frac{3}{4}$. So, $y - 0 = \frac{3}{4}(x - 6) \Rightarrow y = \frac{3}{4}x - \frac{9}{2}$ Tangent line

(d)
$$-\frac{3}{4}x = \frac{3}{4}x - \frac{9}{2}$$
$$\frac{3}{2}x = \frac{9}{2}$$
$$x = 3$$

Intersection: $\left(3, -\frac{9}{4}\right)$

2. Let $y = mx + 1$ be a tangent line to the circle from the point (0, 1). Because the center of the circle is at $(0, -1)$ and the radius is 1 you have the following.

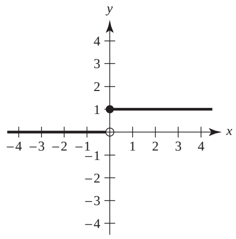
$$x^2 + (y + 1)^2 = 1$$
$$x^2 + (mx + 1 + 1)^2 = 1$$
$$(m^2 + 1)x^2 + 4mx + 3 = 0$$

Setting the discriminant $b^2 - 4ac$ equal to zero,

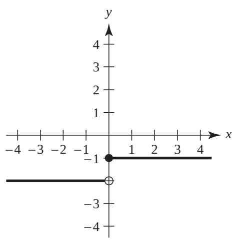
$$16m^2 - 4(m^2 + 1)(3) = 0$$
$$16m^2 - 12m^2 = 12$$
$$4m^2 = 12$$
$$m = \pm\sqrt{3}$$

Tangent lines: $y = \sqrt{3}x + 1$ and $y = -\sqrt{3}x + 1$.

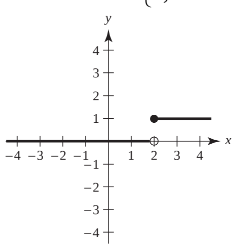
3. $H(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases}$



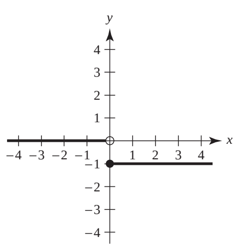
(a) $H(x) - 2 = \begin{cases} -1, & x \geq 0 \\ -2, & x < 0 \end{cases}$



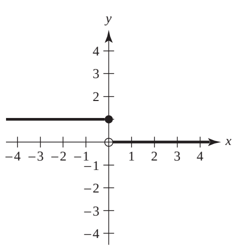
(b) $H(x - 2) = \begin{cases} 1, & x \geq 2 \\ 0, & x < 2 \end{cases}$



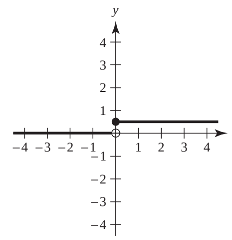
(c) $-H(x) = \begin{cases} -1, & x \geq 0 \\ 0, & x < 0 \end{cases}$



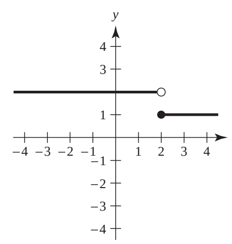
(d) $H(-x) = \begin{cases} 1, & x \leq 0 \\ 0, & x > 0 \end{cases}$



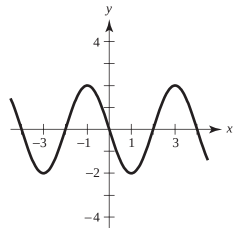
(e) $\frac{1}{2}H(x) = \begin{cases} \frac{1}{2}, & x \geq 0 \\ 0, & x < 0 \end{cases}$



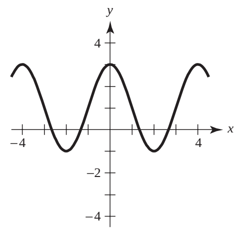
(f) $-H(x - 2) + 2 = \begin{cases} 1, & x \geq 2 \\ 2, & x < 2 \end{cases}$



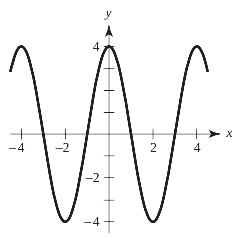
4. (a) $f(x+1)$



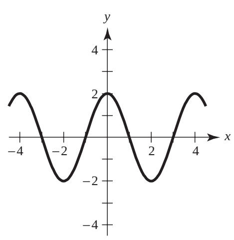
(b) $f(x)+1$



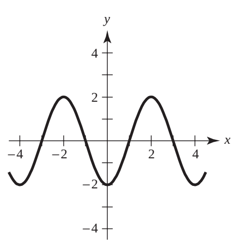
(c) $2f(x)$



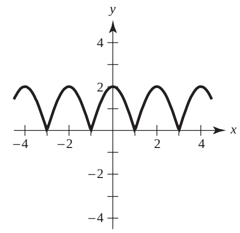
(d) $f(-x)$



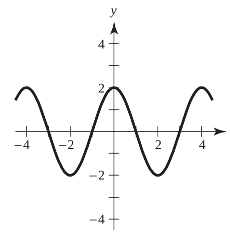
(e) $-f(x)$



(f) $\left|f(x)\right|$



(g) $f(|x|)$

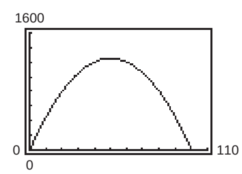


5. (a) $x + 2y = 100 \Rightarrow y = \dfrac{100 - x}{2}$

$$A(x) = xy = x\left(\frac{100 - x}{2}\right) = -\frac{x^2}{2} + 50x$$

Domain: $0 < x < 100$ or $(0, 100)$

(b)



Maximum of 1250 m^2 at $x = 50$ m, $y = 25$ m.

(c) $A(x) = -\frac{1}{2}\left(x^2 - 100x\right)$

$= -\frac{1}{2}\left(x^2 - 100x + 2500\right) + 1250$

$= -\frac{1}{2}(x - 50)^2 + 1250$

$A(50) = 1250$ m^2 is the maximum.

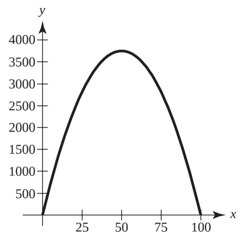
$x = 50$ m, $y = 25$ m

6. (a) $4y + 3x = 300 \Rightarrow y = \dfrac{300 - 3x}{4}$

$A(x) = x(2y) = x\left(\dfrac{300 - 3x}{2}\right) = \dfrac{-3x^2 + 300x}{2}$

Domain: $0 < x < 100$

(b)



Maximum of 3750 ft^2 at $x = 50$ ft, $y = 37.5$ ft.

(c) $A(x) = -\frac{3}{2}(x^2 - 100x)$

$= -\frac{3}{2}(x^2 - 100x + 2500) + 3750$

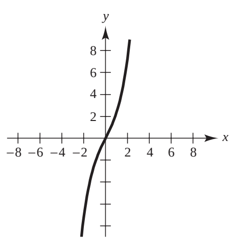
$= -\frac{3}{2}(x - 50)^2 + 3750$

$A(50) = 3750$ square feet is the maximum area, where $x = 50$ ft and $y = 37.5$ ft.

7. The length of the trip in the water is $\sqrt{2^2 + x^2}$, and the length of the trip over land is $\sqrt{1 + (3 - x)^2}$.

So, the total time is $T = \dfrac{\sqrt{4 + x^2}}{2} + \dfrac{\sqrt{1 + (3 - x)^2}}{4}$ hours.

8. $f(x) = e^x = e^{-x}$



$y = e^x - e^{-x}$

$ye^x = e^{2x} - 1$

$(e^x)^2 - ye^x - 1 = 0$ (Quadratic in e^x)

$e^x = \dfrac{y \pm \sqrt{y^2 + 4}}{2}$

$e^x = \dfrac{x + \sqrt{y^2 + 4}}{2}$ (Use positive solution.)

$e^y = \dfrac{x + \sqrt{x^2 + 4}}{2}$

$f^{-1}(x) = y = \ln\left[\dfrac{x + \sqrt{x^2 + 4}}{2}\right]$ Inverse

9. (a) Slope $= \dfrac{9-4}{3-2} = 5$. Slope of tangent line is less than 5.

(b) Slope $= \dfrac{4-1}{2-1} = 3$. Slope of tangent line is greater than 3.

(c) Slope $= \dfrac{4.41-4}{2.1-2} = 4.1$. Slope of tangent line is less than 4.1.

(d) Slope $= \dfrac{f(2+h)-f(2)}{(2+h)-2}$

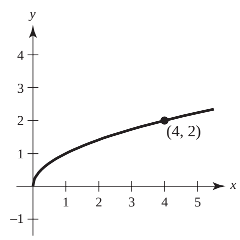
$$= \frac{(2+h)^2-4}{h}$$

$$= \frac{4h+h^2}{h}$$

$$= 4+h,\ h \neq 0$$

(e) Letting h get closer and closer to 0, the slope approaches 4. So, the slope at (2, 4) is 4.

10.



(a) Slope $= \dfrac{3-2}{9-4} = \dfrac{1}{5}$. Slope of tangent line is greater than $\dfrac{1}{5}$.

(b) Slope $= \dfrac{2-1}{4-1} = \dfrac{1}{3}$. Slope of tangent line is less than $\dfrac{1}{3}$.

(c) Slope $= \dfrac{2.1-2}{4.41-4} = \dfrac{10}{41}$. Slope of tangent line is greater than $\dfrac{10}{41}$.

(d) Slope $= \dfrac{f(4+h)-f(4)}{(4+h)-4} = \dfrac{\sqrt{4+h}-2}{h}$

(e) $\dfrac{\sqrt{4+h}-2}{h} = \dfrac{\sqrt{4+h}-2}{h} \cdot \dfrac{\sqrt{4+h}+2}{\sqrt{4+h}+2} = \dfrac{(4+h)-4}{h\left(\sqrt{4+h}+2\right)} = \dfrac{1}{\sqrt{4+h}+2},\ h \neq 0$

As h gets closer to 0, the slope gets closer to $\dfrac{1}{4}$. The slope is $\dfrac{1}{4}$ at the point (4, 2).

11. $f(x) = y = \dfrac{1}{1-x}$

(a) Domain: all $x \neq 1$ or $(-\infty, 1) \cup (1, \infty)$

Range: all $y \neq 0$ or $(-\infty, 0) \cup (0, \infty)$

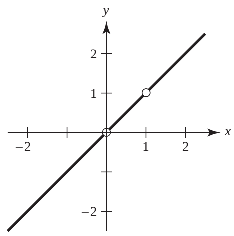
(b) $f(f(x)) = f\left(\dfrac{1}{1-x}\right) = \dfrac{1}{1-\left(\dfrac{1}{1-x}\right)} = \dfrac{1}{\dfrac{1-x-1}{1-x}} = \dfrac{1-x}{-x} = \dfrac{x-1}{x}$

Domain: all $x \neq 0, 1$ or $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

(c) $f(f(f(x))) = f\left(\dfrac{x-1}{x}\right) = \dfrac{1}{1-\left(\dfrac{x-1}{x}\right)} = \dfrac{1}{\dfrac{1}{x}} = x$

Domain: all $x \neq 0, 1$ or $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

(d) The graph is not a line. It has holes at (0, 0) and (1, 1).



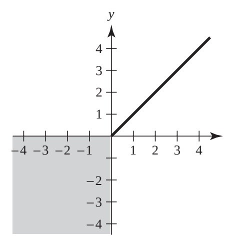
12. Using the definition of absolute value, you can rewrite the equation.

$$y + |y| = x + |x|$$

$$\begin{cases} 2y, & y > 0 \\ 0, & y \leq 0 \end{cases} = \begin{cases} 2x, & x > 0 \\ 0, & x \leq 0 \end{cases}.$$

For $x > 0$ and $y > 0$, you have $2y = 2x \Rightarrow y = x$.

For any $x \leq 0$, y is any $y \leq 0$. So, the graph of $y + |y| = x + |x|$ is as follows.



13. (a) $\dfrac{I}{x^2} = \dfrac{2I}{(x-3)^2}$

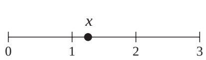
$$x^2 - 6x + 9 = 2x^2$$

$$x^2 + 6x - 9 = 0$$

$$x = \frac{-6 \pm \sqrt{36 + 36}}{2}$$

$$= -3 \pm \sqrt{18}$$

$$\approx 1.2426, \ -7.2426$$



(b) $\dfrac{I}{x^2 + y^2} = \dfrac{2I}{(x-3)^2 + y^2}$

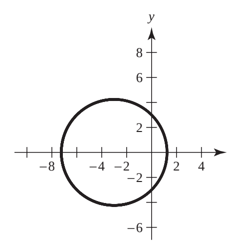
$$(x-3)^2 + y^2 = 2(x^2 + y^2)$$

$$x^2 - 6x + 9 + y^2 = 2x^2 + 2y^2$$

$$x^2 + y^2 + 6x - 9 = 0$$

$$(x+3)^2 + y^2 = 18$$

Circle of radius $\sqrt{18}$ and center $(-3, 0)$.



14. (a) $\dfrac{I}{x^2 + y^2} = \dfrac{kI}{(x-4)^2 + y^2}$

$$(x-4)^2 + y^2 = k(x^2 + y^2)$$

$$(k-1)x^2 + 8x + (k-1)y^2 = 16$$

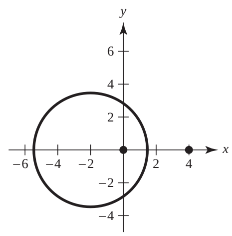
If $k = 1$, then $x = 2$ is a vertical line. Assume $k \neq 1$.

$$x^2 + \frac{8x}{k-1} + y^2 = \frac{16}{k-1}$$

$$x^2 + \frac{8x}{k-1} + \frac{16}{(k-1)^2} + y^2 = \frac{16}{k-1} + \frac{16}{(k-1)^2}$$

$$\left(x + \frac{4}{k-1}\right)^2 + y^2 = \frac{16k}{(k-1)^2}, \text{ Circle}$$

(b) If $k = 3$, $(x + 2)^2 + y^2 = 12$

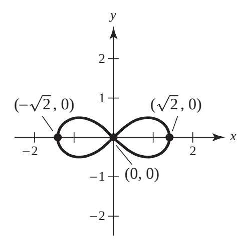


(c) As k becomes very large, $\dfrac{4}{k - 1} \to 0$ and $\dfrac{16k}{(k - 1)^2} \to 0$.

The center of the circle gets closer to (0, 0), and its radius approaches 0.

15.

$$\begin{aligned} d_1 d_2 &= 1 \\ \left[(x + 1)^2 + y^2\right]\left[(x - 1)^2 + y^2\right] &= 1 \\ (x + 1)^2(x - 1)^2 + y^2\left[(x + 1)^2 + (x - 1)^2\right] + y^4 &= 1 \\ (x^2 - 1)^2 + y^2\left[2x^2 + 2\right] + y^4 &= 1 \\ x^4 - 2x^2 + 1 + 2x^2y^2 + 2y^2 + y^4 &= 1 \\ (x^4 + 2x^2y^2 + y^4) - 2x^2 + 2y^2 &= 0 \\ (x^2 + y^2)^2 &= 2(x^2 - y^2) \end{aligned}$$



Let $y = 0$. Then $x^4 = 2x^2 \Rightarrow x = 0$ or $x^2 = 2$.

So, $(0, 0)$, $(\sqrt{2}, 0)$ and $(-\sqrt{2}, 0)$ are on the curve.